Cross-Layer Design of All-Optical Networks Incorporating Crosstalk Effects

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Overview

- ▷ Introduction
- ▷ Crosstalk Propagation in All-Optical Networks
- ▷ Cross-Layer Adaptive Routing and Wavelength Assignment in All-Optical Networks
- ▷ Analysis of Blocking Probability in Crosstalk-Impaired Networks
- ▷ Conclusions

Publications
Crosstalk Propagation in All-Optical Networks

- Y. Pointurier and M. Brandt-Pearce, "Study of Crosstalk Enhancement by Fiber Nonlinearity in All-Optical Networks using a Novel Semi-Analytical Method", submitted to *IEEE Communication Letters*.
- Y. Pointurier and M. Brandt-Pearce, "Analytical Study of Crosstalk Propagation in All-Optical Networks Using Perturbation Theory," *IEEE Journal of Lightwave Technology*, Dec. 2005.
- Y. Pointurier and M. Brandt-Pearce, "Study of Crosstalk Propagation in All-Optical Networks using Perturbation Theory," *IEEE International Conference on Electronics, Circuits and Systems (ICECS)*, Dec. 2005.
- Y. Pointurier and M. Brandt-Pearce, "Effects of Crosstalk on the Performance and Design of All-Optical Networks with Fiber Nonlinearities," *Asilomar Conference on Signals, Systems & Computers*, Nov. 2004.

Publications
Cross-Layer Adaptive RWA in All-Optical Networks

- Y. Pointurier, M. Brandt-Pearce, S. Subramaniam, and B. Xu, "Cross-Layer Adaptive Routing and Wavelength Assignment in All-Optical Networks," submitted to *IEEE Journal on Selected Areas in Communications*.
- Y. Pointurier, M. Brandt-Pearce, and B. Xu, "Reduction of Crosstalk in WDM Networks With Optional Coding," *IASTED International Conference on Optical Communication Systems and Networks (OCSN)*, Jul. 2006.
- Y. Pointurier, M. Brandt-Pearce, T. Deng, and S. Subramaniam, "Fair QoS-Aware Adaptive Routing and Wavelength Assignment in All-Optical Networks," *IEEE ICC*, Jun. 2006. **(Symposium best paper award.)**
- Y. Pointurier, M. Brandt-Pearce, T. Deng and S. Subramaniam, "Fair Routing and Wavelength Assignment in All-Optical Networks," *IEEE/OSA OFC*, Mar. 2006.
- Y. Pointurier and M. Brandt-Pearce, "Routing and Wavelength Assignment Incorporating the Effects of Crosstalk Enhancement by Fiber Nonlinearity," *Conference on Information Sciences and Systems (CISS)*, Mar. 2005.

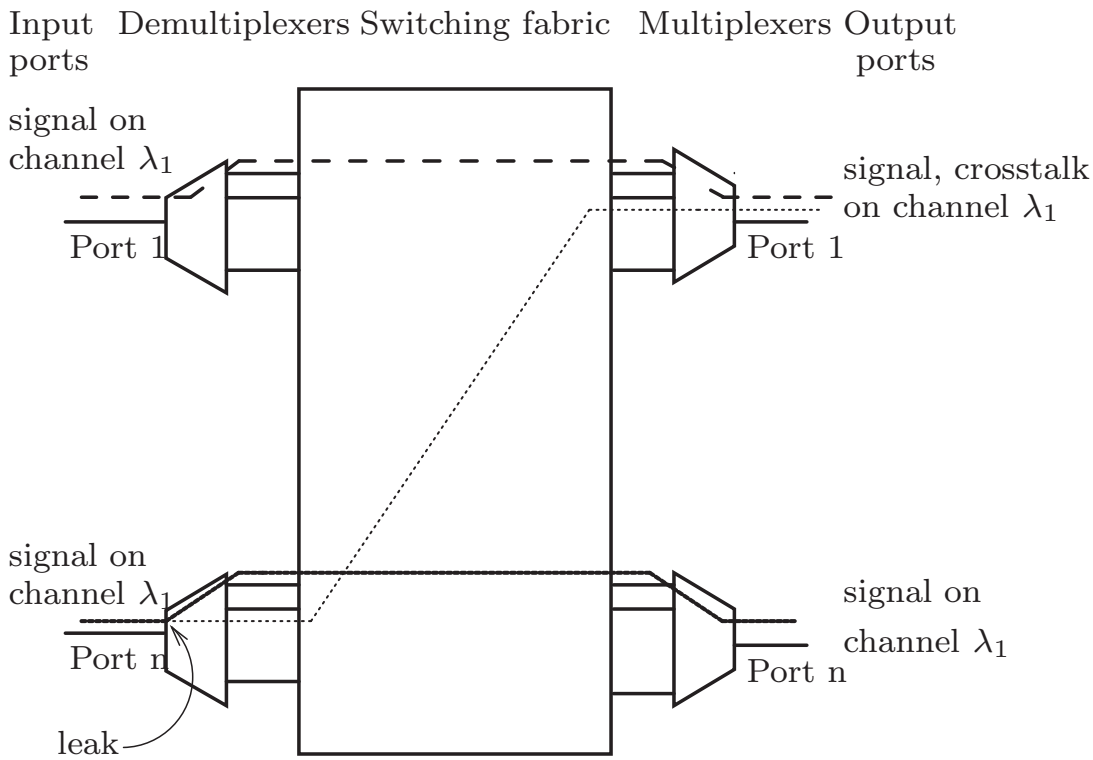
Publications
Analysis of Blocking Probability in Crosstalk-Impaired Networks

- Y. Pointurier, M. Brandt-Pearce, and S. Subramaniam, "Analysis of Blocking Probability in Noise and Crosstalk Impaired All-Optical Networks," submitted to *IEEE INFOCOM 2007*.

All-optical networks

- Current high-speed optical networks
  - Bottleneck due to electrical conversions
- Features of all-optical networks
  - Speed, flexibility, cost
- New issues arise with all-optical networks
  - Nodes (OXCs) are subject to crosstalk
  - Node crosstalk is transmitted over extremely long paths without electrical signal regeneration → nonlinear interaction
- Implementation
  - Crosstalk issues will be encountered in the near future

Node crosstalk

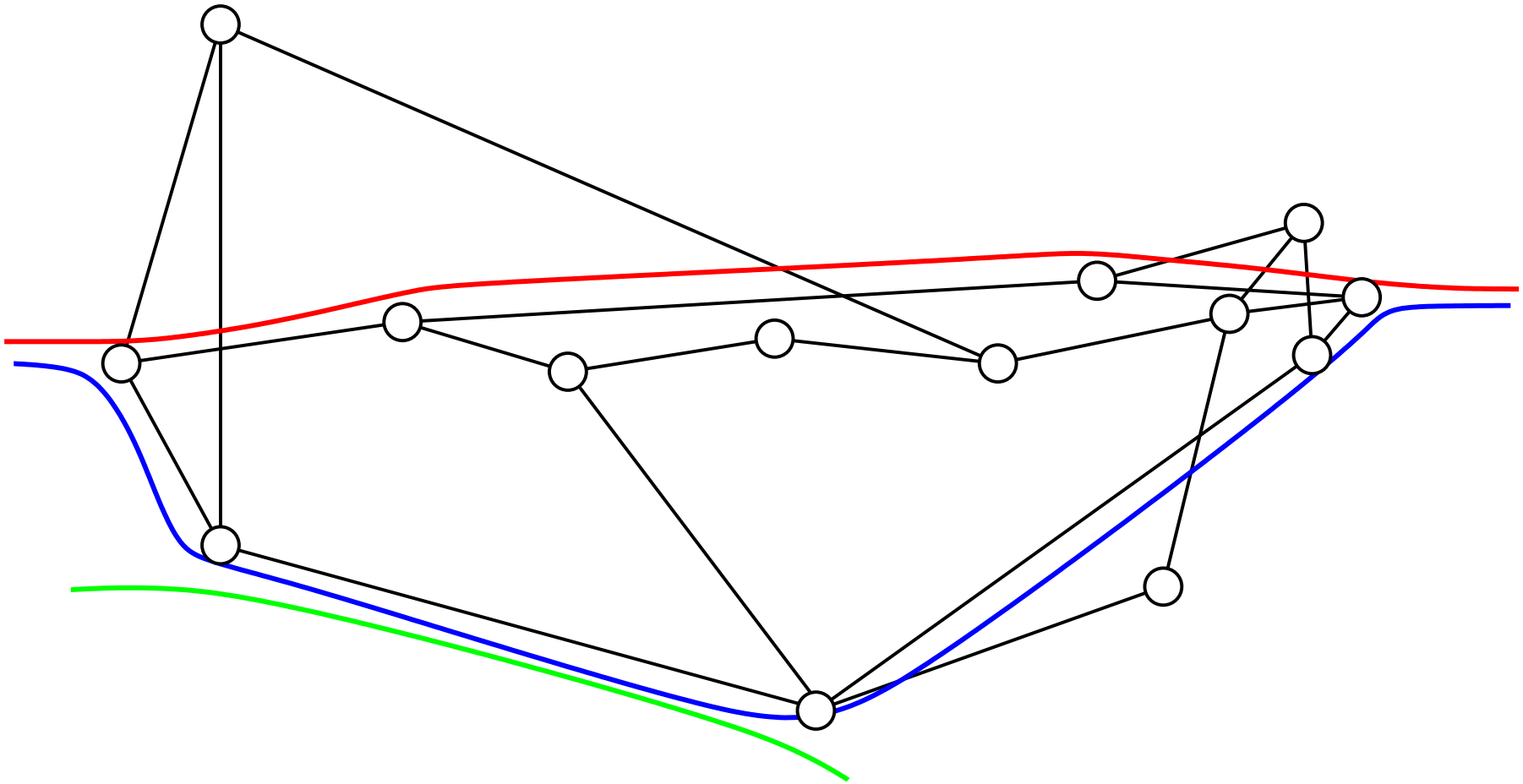


- ▷ Leaks can originate from imperfect demultiplexing, or transmission within the switching matrix of an all-optical crossconnect (OXC)

Routing and Wavelength Assignment

- All-optical networks with no wavelength conversion
- Crosstalk impairs the QoS of lightpaths
  - typically we want to keep $BER \leq 10^{-9}$ at all times, for each lightpath
- Design Routing and Wavelength Assignment (RWA) algorithms to mitigate crosstalk effects

Routing and Wavelength Assignment illustration



Routing and Wavelength Assignment

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  - typically we want to keep $BER \leq 10^{-9}$ at all times, for each lightpath
- Design Routing and Wavelength Assignment (RWA) algorithms to mitigate crosstalk effects
  - wavelength continuity constraint and QoS constraint
  - traditional RWA try to decrease call blocking probability
  - adaptive RWA is a class of RWA with low blocking probability due to the wavelength continuity constraint $\Rightarrow$ focus on QoS blocking and fairness among users

Contributions

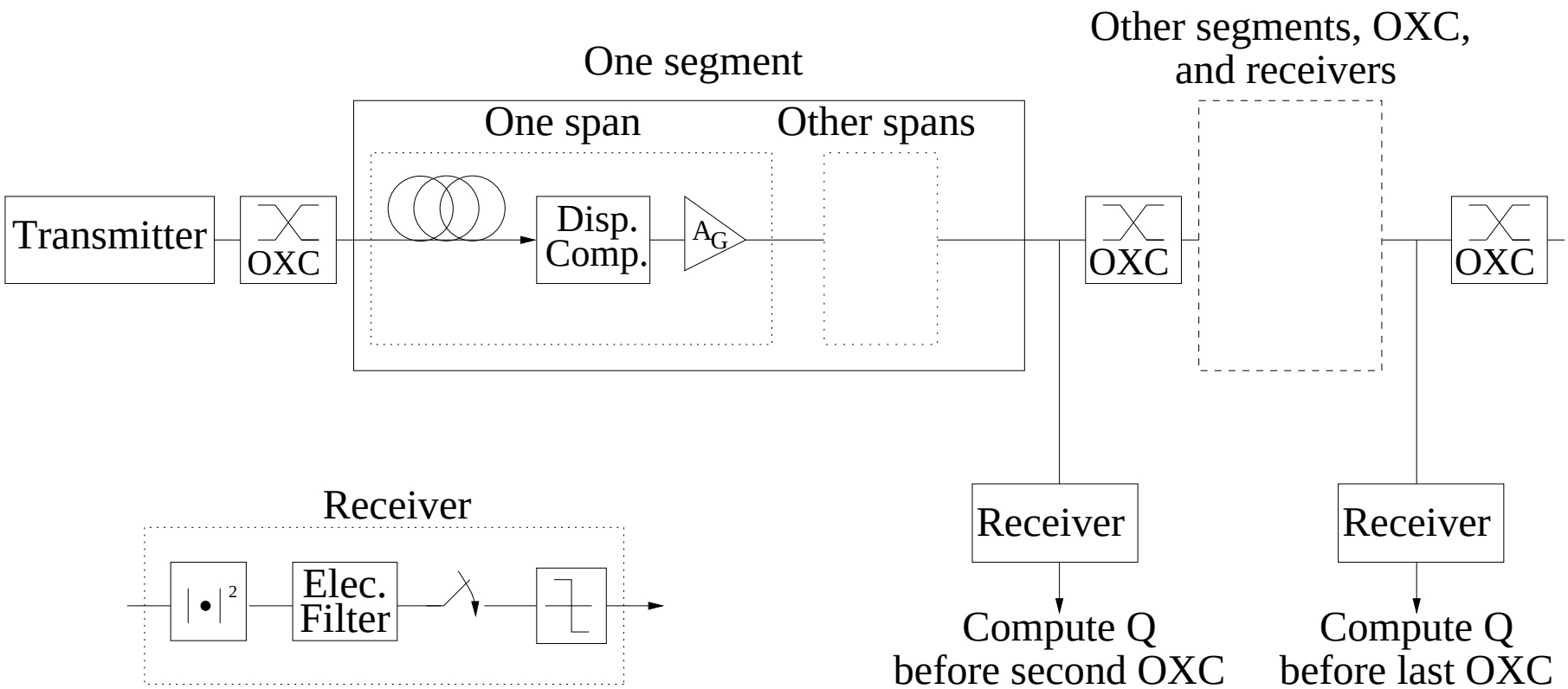
Cross-layer design: mitigate **physical-layer** effects (including crosstalk) at the **network layer** (with RWA)

- ▷ Fast analytical and semi-analytical models to determine performance (BER) of all-optical networks subject to crosstalk
- ▷ Novel RWA algorithms accounting for physical layer impairments (ISI, noise, crosstalk)
- ▷ Analysis of a class of RWA algorithms incorporating physical layer impairments

Overview

- Introduction
- **Crosstalk Propagation in All-Optical Networks**
  - System description
  - Continuous wave approximation (NRZ signals) and validation by simulations
  - Semi-analytical method (RZ signals) and validation by simulations
- Cross-Layer Adaptive Routing and Wavelength Assignment in All-Optical Networks
- Analysis of Blocking Probability in Crosstalk-Impaired Networks
- Conclusions

Lightpath and physical layer models



$$BER = \frac{1}{2}\mathrm{erfc}\frac{Q}{\sqrt{2}}, \quad Q = \frac{\mu_1 - \mu_0}{\sigma_0 + \sqrt{\sigma_i^2 + \sigma_n^2 + \sum \sigma_x^2}}$$

Problem statement

- **What is the impact of node crosstalk and how does it depend on the physical parameters ?**
  - $Q = \frac{\mu_1 - \mu_0}{\sigma_0 + \sqrt{\sigma_i^2 + \sigma_n^2 + \sum \sigma_x^2}}$
  - Simulations are very slow (tens of hours) because of the number of random parameters
  - Can compute μ_0, μ_1, σ_0, σ_i using very short simulations of a few bits (ISI, linear and nonlinear signal transmission effects)
  - Can compute σ_n due to noise using analytical techniques
  - Need a fast way to compute σ_x and hence Q

Node crosstalk model

- $s_0(t) = s_m(t) + \sum_{\ell} m_\ell g_0(t - \ell T_b)$
  - **in the analysis only**, $s_m(t)$ is assumed to be a Continuous Wave (CW): $s_m(t) = \sqrt{P_0}$
  - $s_m(t)$ is modulated in the simulations
  - Node crosstalk signal is modulated (analysis and simulations)
- $g_0(t) = \sqrt{\eta P_0} h(t - \tau) e^{j\omega_s (t-\tau) + j\varphi}$
  - Bits m_ℓ, delay τ and phase φ are uniformly distributed over $\{0,1\}$, $[0,T_b)$, $[0, 2\pi)$, respectively.
  - Node crosstalk attenuation η, detuning ω_s
  - Pulse shape $h(t)$

Analysis: determining a transfer matrix

- CW input, after k spans:

$$s_m(t, L) = e^{jk\theta_{SPM}} \sqrt{P_0} \quad \text{with} \quad (\theta_{SPM} \approx -\gamma P_0/\alpha)$$

- Node crosstalk bit, after k spans:

$$g_k(t) = e^{jk\theta_{SPM}} (g_k^I(t) + j g_k^Q(t))$$

- Projecting:

$$g_k^I(t) = g_k^{I+}(t) e^{j\varphi} + g_k^{I-}(t) e^{-j\varphi}$$
$$g_k^Q(t) = g_k^{Q+}(t) e^{j\varphi} + g_k^{Q-}(t) e^{-j\varphi}$$

Continued ...

- In frequency domain, for the first span (same for I-/Q- terms):

$$\begin{bmatrix} G_1^{I+}(\omega) \\ G_1^{Q+}(\omega) \end{bmatrix} = T_1(\omega) \begin{bmatrix} G_0^{I+}(\omega) \\ G_0^{Q+}(\omega) \end{bmatrix}$$

- After k spans (same for I-/Q- terms):

$$\begin{bmatrix} G_k^{I+}(\omega) \\ G_k^{Q+}(\omega) \end{bmatrix} = T_k(\omega) \ldots T_1(\omega) \begin{bmatrix} G_0^{I+}(\omega) \\ G_0^{Q+}(\omega) \end{bmatrix}$$

Node crosstalk variance

- Current due to node crosstalk power after k spans (single segment, receiver noise ignored):

$$\Delta i(t,k) = \rho f(t) * \left(\left| \sqrt{P_0} e^{jk\theta_{SPM}} + m g_k(t-\tau) \right|^2 - P_0 \right)$$

$$\Rightarrow \Delta i(t,k) \approx 2\rho\sqrt{P_0} m f(t) * \left(g_k^{I+}(t-\tau) e^{j\varphi} + g_k^{I-}(t-\tau) e^{-j\varphi} \right)$$

- $\sigma_x^2(t,k)$ is the variance of $\Delta i(t,k)$ sampled at $t = T_b/2$

$$\sigma_x^2 = 4\rho^2 P_0 \int_{-\infty}^{\infty} \frac{1}{T_b} |f(t) * g_k^{I+}(t-\tau)|^2 \mathrm{d}\tau$$

Node crosstalk variance: continued

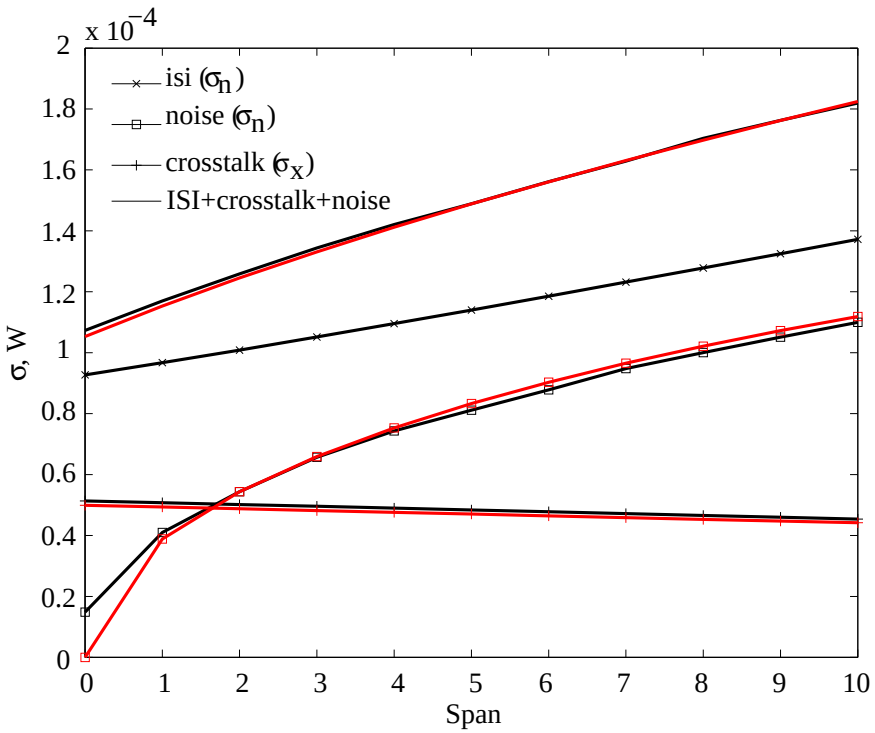
$$\sigma_x^2 = 4\rho^2 P_0 \int_{-\infty}^{\infty} \frac{1}{T_b} |f(t) * g_k^{I+}(t-\tau)|^2 \mathrm{d}\tau$$

- Multisegment case: add node crosstalk variances for each node crosstalk component coming from a different OXC
- Complexity:
  - analysis is simpler than even the short simulation
  - short simulation: 32 bits
  - complete simulation: 2048 x 32 bits

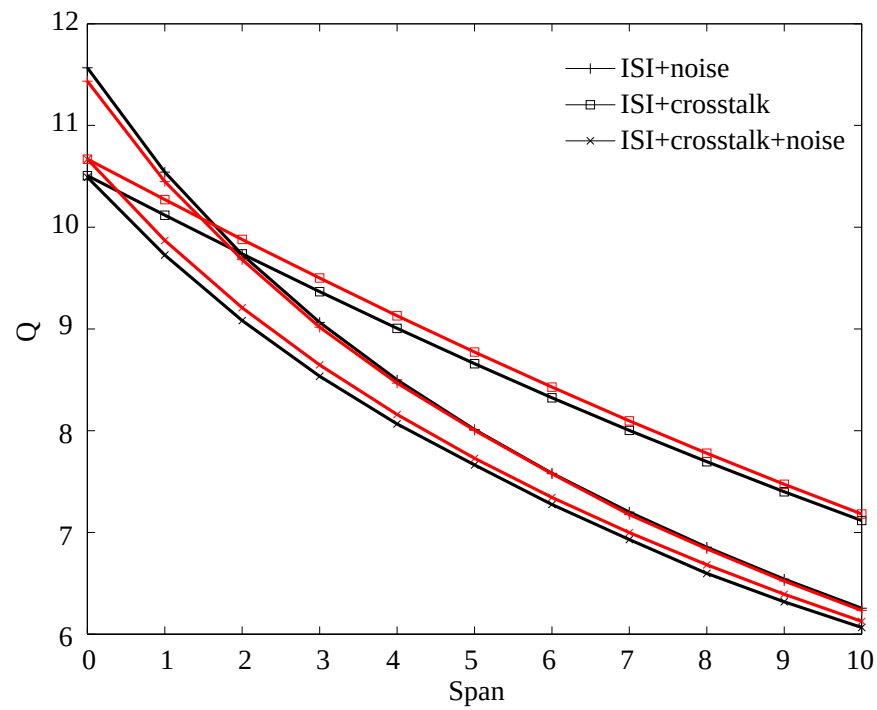
Validation: physical parameters

Description	Baseline Value
Span length	100 km
Pump power	2 mW
Bit rate	10 Gbps
Pulse shape	NRZ
Node crosstalk attn. (power)	−30 dB
Node crosstalk detuning	0 GHz
Fiber loss	0.2 dB/km
Nonlinear coefficient	2.2 $(\mathrm{W\ km})^{-1}$
2nd order dispersion	17 ps/nm/km
Noise figure	2
Dispersion compensation	100% post
Photodetector responsitivity	1 A/W (arbitrarily)
Electrical filter bandwidth	0.7× bit rate

10 Gbps dispersion-compensated network



Standard deviations



Q factors

red: analysis; **black**: simulation

Semi-analytical method for RZ signals

- With RZ signaling, the CW assumption no longer holds
- Recall:

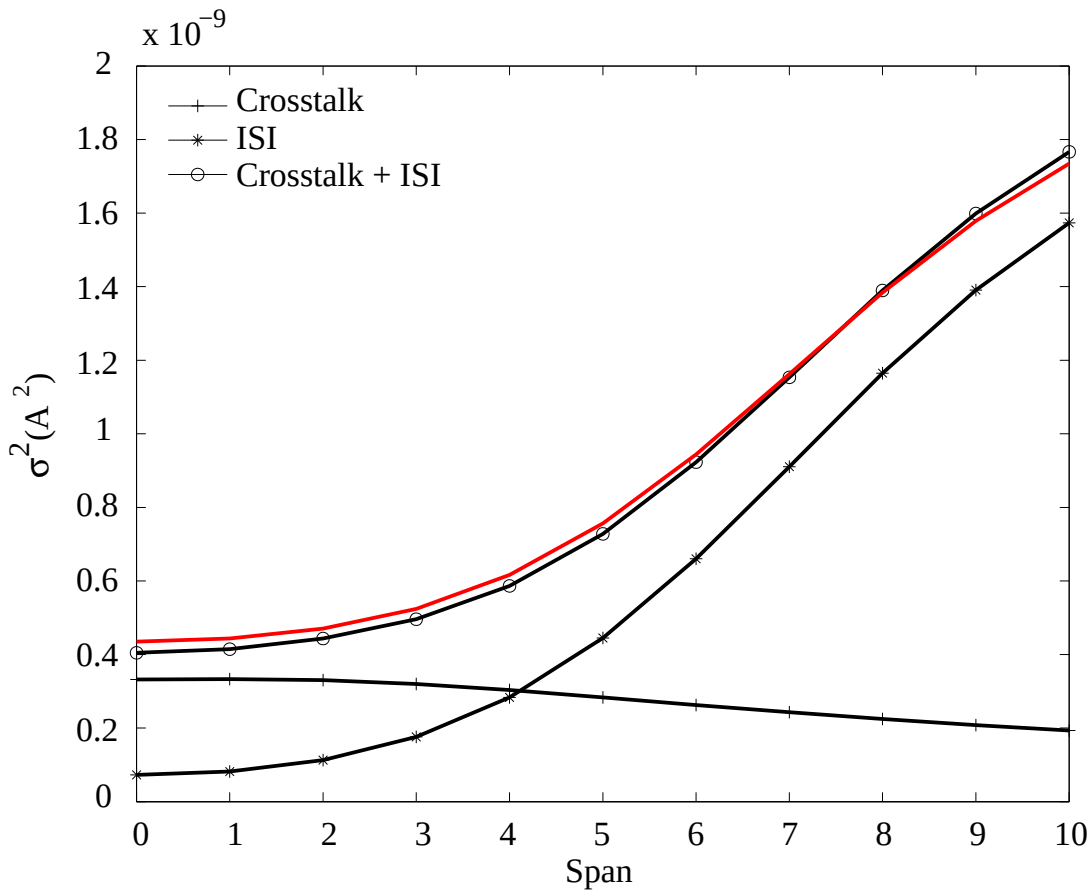
$$\sigma_x^2(t) = 4\rho^2 \int_{-\frac{T_b}{2}}^{\frac{T_b}{2}} \frac{1}{T_b} \left| f(t) * \left(\sqrt{P(t)} g_\tau^{I+}(t) \right) \right|^2 \mathrm{d}\tau$$

- We determine $g_\tau^{I+}(t)|_{t=T_b/2}$ for all τ by simulation

Validation: physical parameters

Description	Baseline value
Span length	50 km
Pump power	2 mW
Bit rate	10 Gbps
Pulse shape	RZ
Node crosstalk attn.	-30 dB
Node crosstalk detuning	0 GHz
Fiber loss	0.2 dB/km
Nonlinear coefficient	2.2 $(\mathrm{W\ km})^{-1}$
Second order dispersion	2 ps/nm/km
Noise figure	no noise
Dispersion compensation	no DC
Photodetector responsitivity	1 A/W
Electrical filter bandwidth	$0.7\times$ bit rate

Impact of ISI and node crosstalk



red: analysis; **black**: simulation

Conclusions: modeling of node crosstalk effects

- ▷ Able to assess accurately Q over broad ranges of physical parameters
- ▷ Node crosstalk effect cannot be ignored
- ▷ Node crosstalk effect may not be constant along a lightpath
- ▷ Need to account for node crosstalk impairment in RWA algorithm design

Overview

- ▷ Introduction
- ▷ Crosstalk Propagation in All-Optical Networks
- ▷ **Cross-Layer Adaptive Routing and Wavelength Assignment in All-Optical Networks**
  - System description
  - Fair QoS-aware adaptive RWA
  - QoS-aware adaptive RWA with optional coding
- ▷ Analysis of Blocking Probability in Crosstalk-Impaired Networks
- ▷ Conclusions

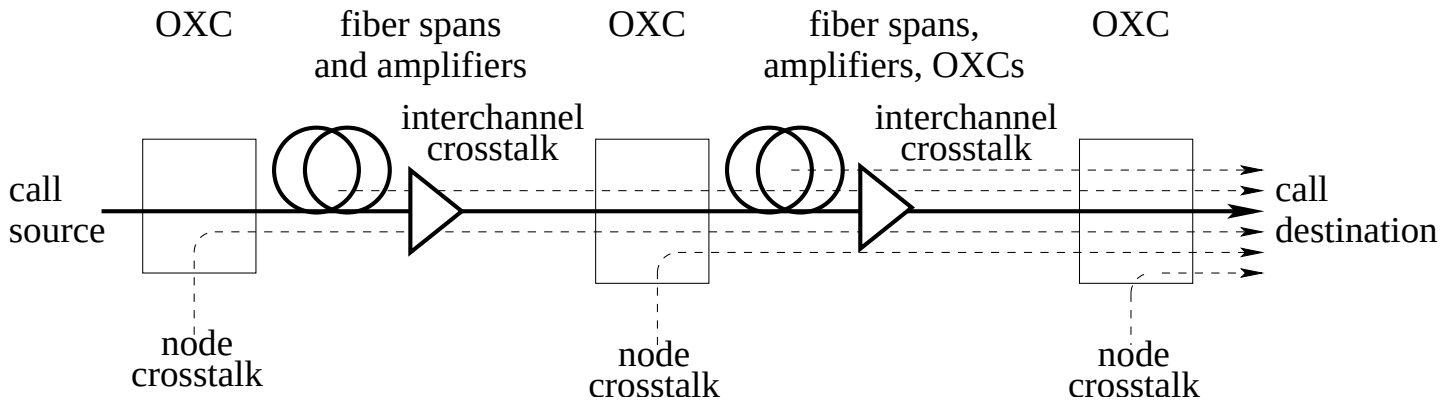
Cross-Layer Adaptive RWA in All-Optical Networks

- RWA: find a lightpath (route + wavelength) between 2 nodes on call arrival
  - wavelength continuity constraint
- Typically designed to minimize call blocking probability
- Design Routing and Wavelength Assignment (RWA) algorithms to mitigate physical-layer effects
  - QoS constraint – node crosstalk model [Deng/Subramaniam-Broadnets2004]
  - adaptive RWA is a class of RWA with low blocking probability due to the wavelength continuity constraint [Mokhtar/Azizoğlu-ToN1998]

Metrics

- Average call blocking probability (BP): keep low
- Bit-error rate (BER)
  - a margin allows for greater scalability, flexibility, robustness, fewer retransmissions at higher layers
- Fairness: $0 \leq f(X) = E_S[X]^2/E_S[X^2] \leq 1$ [Jain1984]
  - here, S is the set of all (source, destination) pairs and X is either BP or BER
- BP fairness: all clients should have equal access to the network
- BER fairness: more fair $\Rightarrow$ need FEC for fewer paths

BER estimator



- ▷ Fast, dynamic estimation of BER: $BER = 0.5\ \mathrm{erfc}(Q/\sqrt{2})$

$$Q = \frac{\mu_1 - \mu_0}{\sigma_0 + \sigma_1} = \frac{\mu_1 - \mu_0}{\sigma_0 + \sqrt{\sigma_i^2 + \sigma_n^2 + \sum_p \sigma_{ix_p}^2 + \sum_q \sigma_{nx_q}^2}}$$

- ▷ μ_0, μ_1, σ_0, σ_1: means, st.dev. for the received "0s" and "1s"
- ▷ Split σ_1^2 into ISI, ASE noise, interchannel and node crosstalk variances
- ▷ Sum crosstalk terms over all interfering paths

Impact of crosstalk

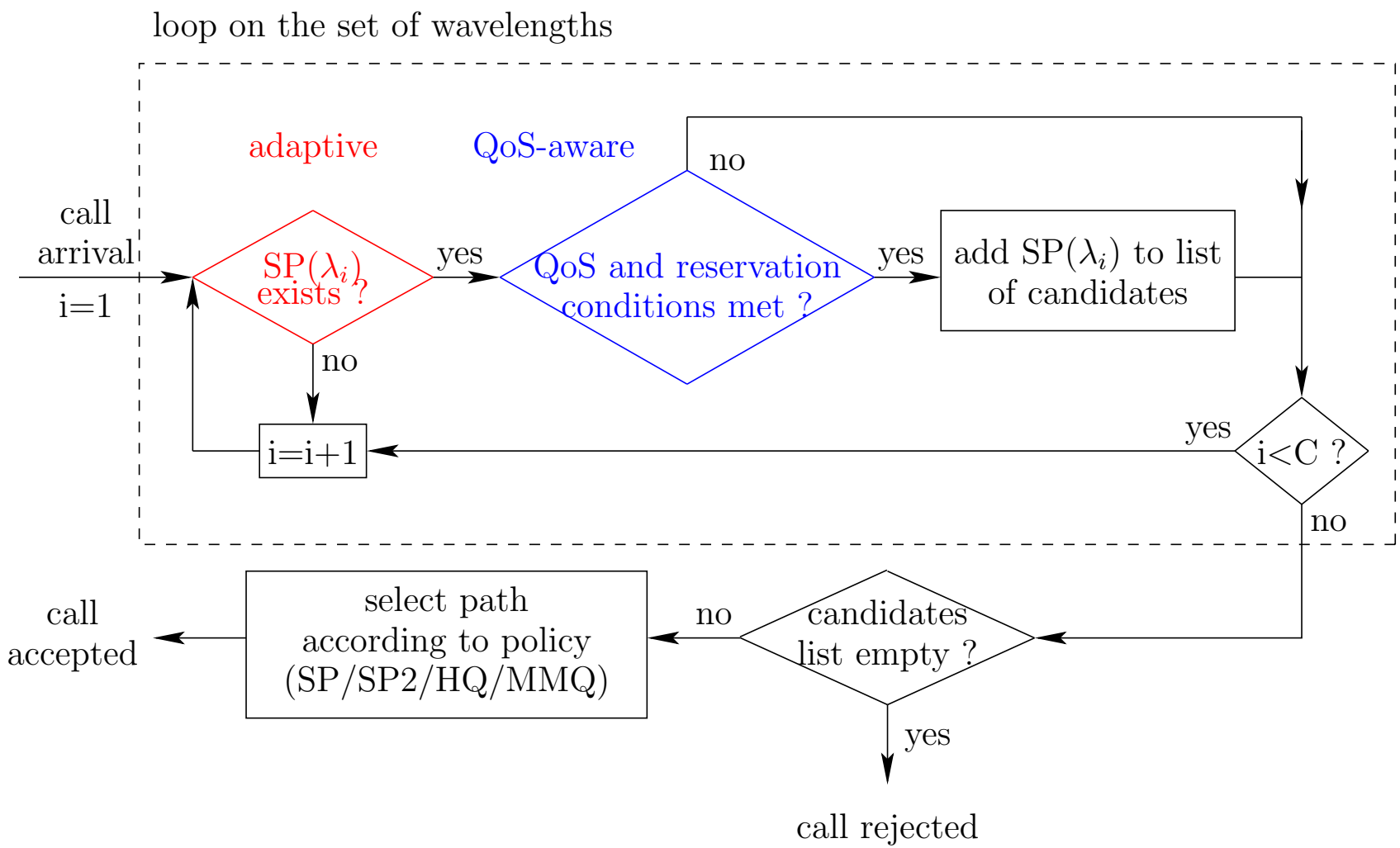
- Impact of crosstalk accumulation on the maximum transmission distance (70-km spans), for a typical optical network
  - node crosstalk

Crosstalks	0, 1	2, 3	4	5, 6	7, 8	9	10
Spans	12	11	10	9	8	7	6

  - XPM

Crosstalks	0	1	2-8
Spans	12	9	7

Generic QoS-aware adaptive RWA algorithm



Reference policy: SP [Mokhtar/Azizoğlu-ToN1998]

- Considered algorithms are as complex as traditional adaptive RWA with BER guarantee
- SP is the traditional Shortest Path policy

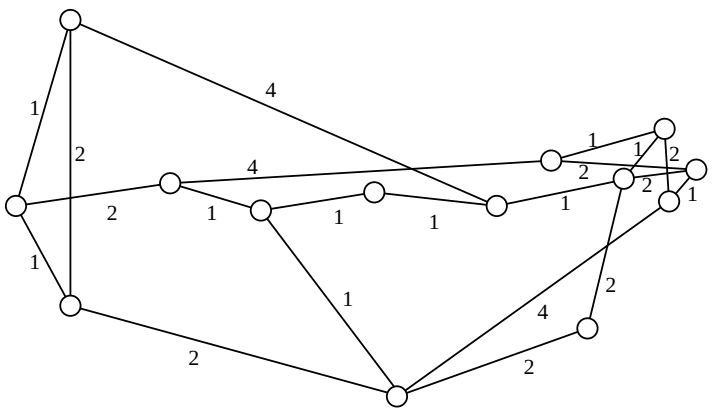
Novel policies: HQ, MMQ

- HQ selects the lightpath with the highest Q factor
- MMQ: inserting a new lightpath "LP1" in the network changes the BER for all LPs that cross LP1; MMQ maximizes (over the set of wavelengths) the minimum Q (over the set of LPs that cross each candidate lightpath)
- Insight: optimize QoS by selecting the path that manages the largest QoS (HQ) or QoS margin (MMQ) in the network

Novel policies: SP2, MMQ2

- SP2 is SP with protecting threshold – single-hop paths accepted only if 2 or more wavelengths are available
- Insight: put aside wavelengths for longer paths, which are more likely to be blocked due to both wavelength continuity and QoS constraints
- MMQ2 additionally implements the protecting threshold technique
- Insight: optimize QoS by selecting the path that manages the largest QoS margin in the network
- HQ/MMQ/MMQ2 are QoS-enhanced but may waste resources compared with SP/SP2

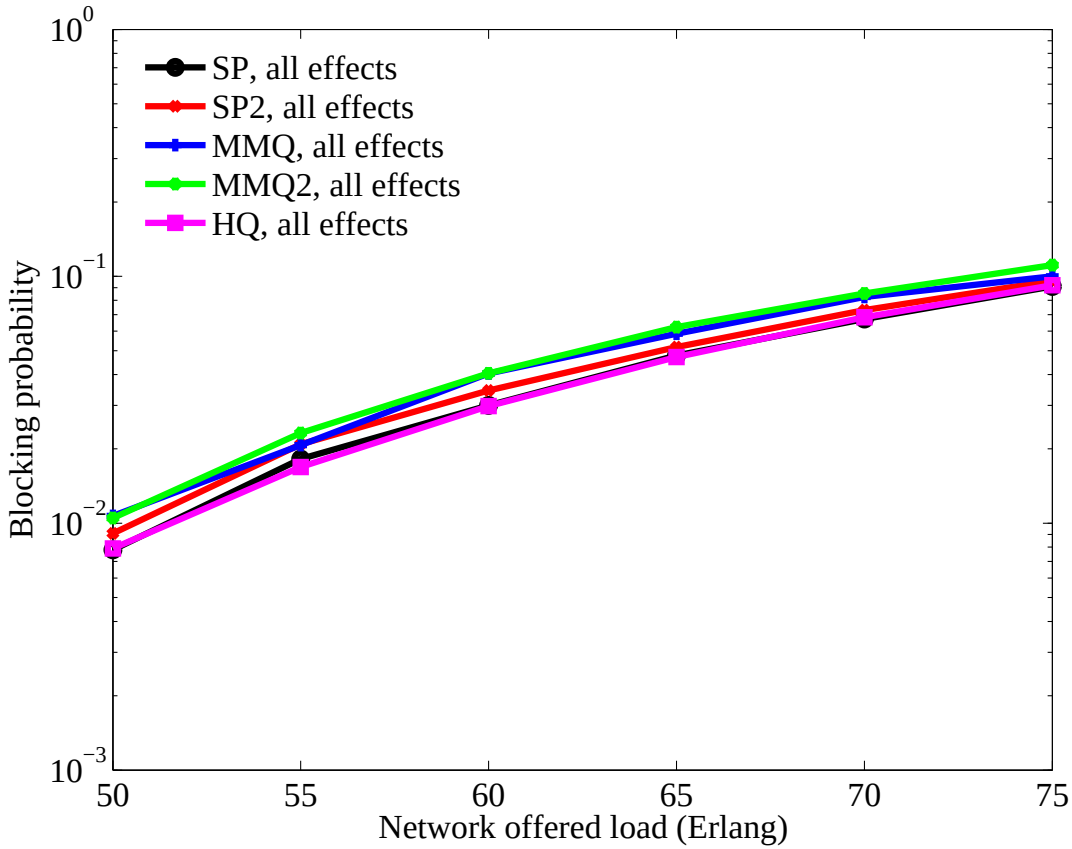
Topology and physical parameters



NSF topology downscaled 10 times; the weights are the number of spans.

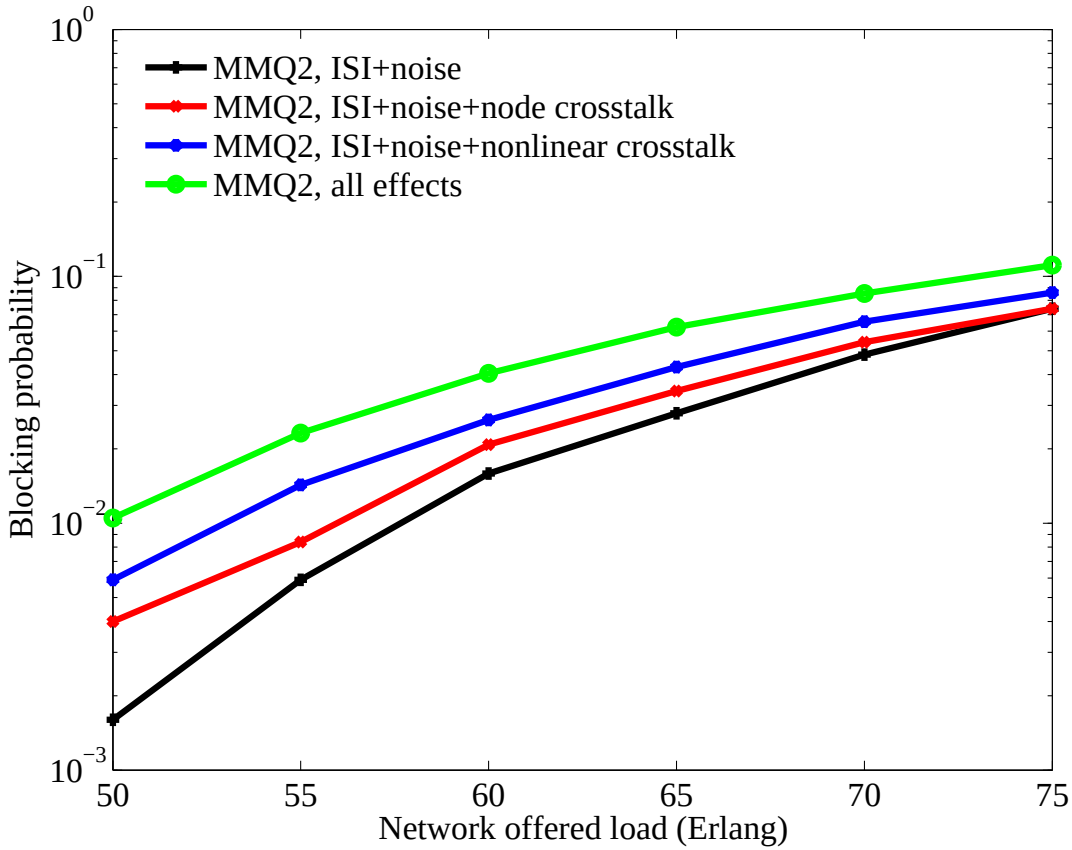
Description	Value
Span length	70 km
Signal peak power	2 mW
Bit rate	10 Gbps
Pulse shape	NRZ
Fabric crosstalk	−40 dB
Adj. port crosstalk	−30 dB
Non adj. port crosstalk	−60 dB
Fiber type	SMF
Dispersion compensation	100% post-DC
Noise factor	2
Receiver elec. BW	7 GHz
Number of WL	8
Maximum BER	10^{-9}

Blocking Probability



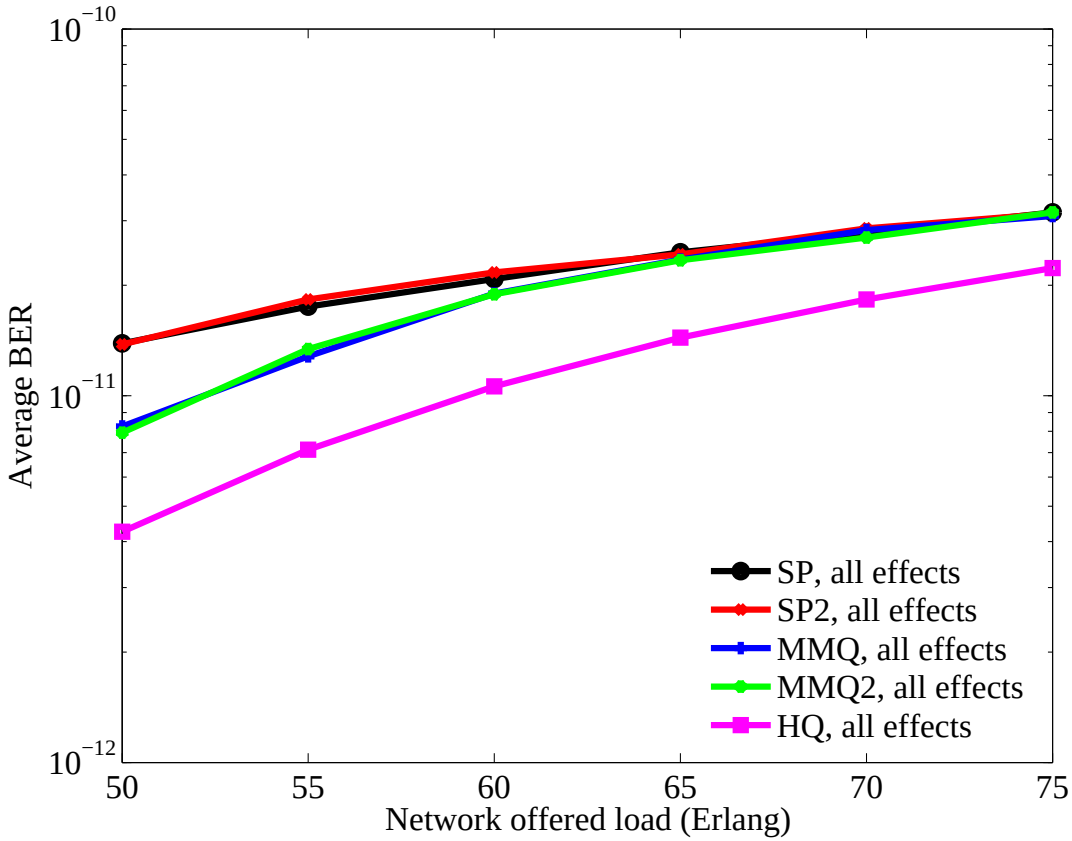
▷ BPs are equivalent for all algorithms

Blocking probability for MMQ2



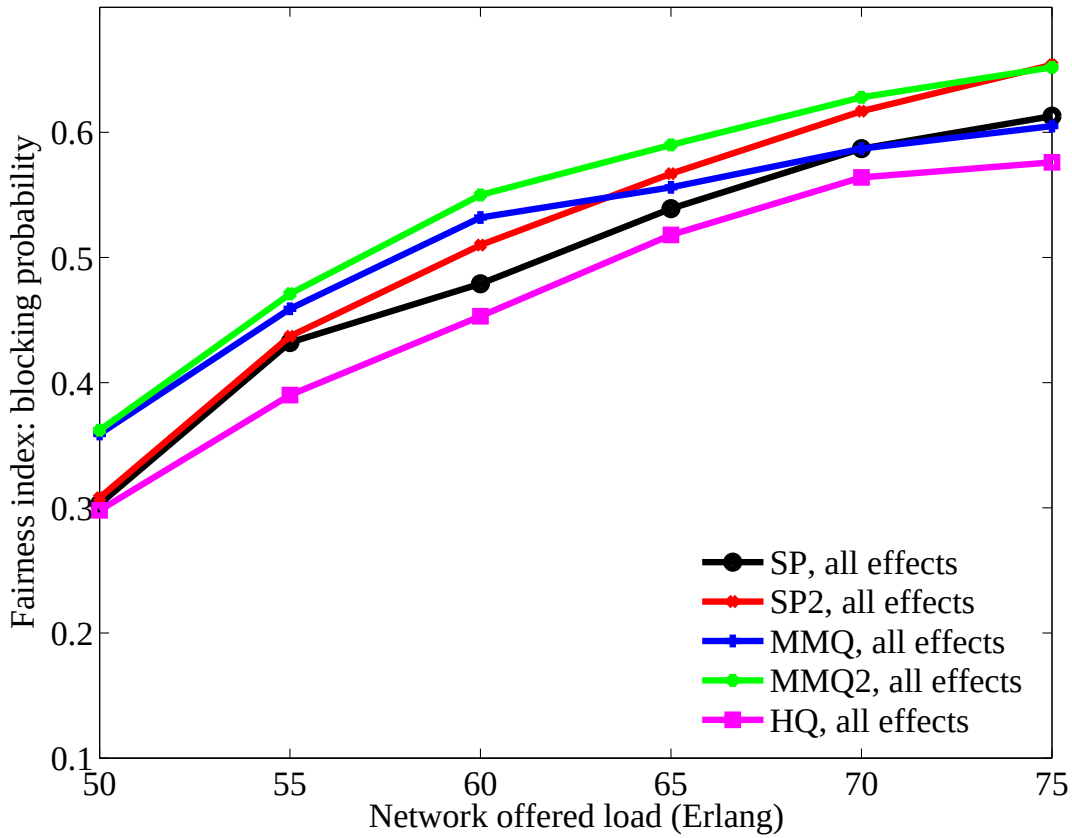
▷ Both kinds of crosstalk have a serious impact on BP

BER



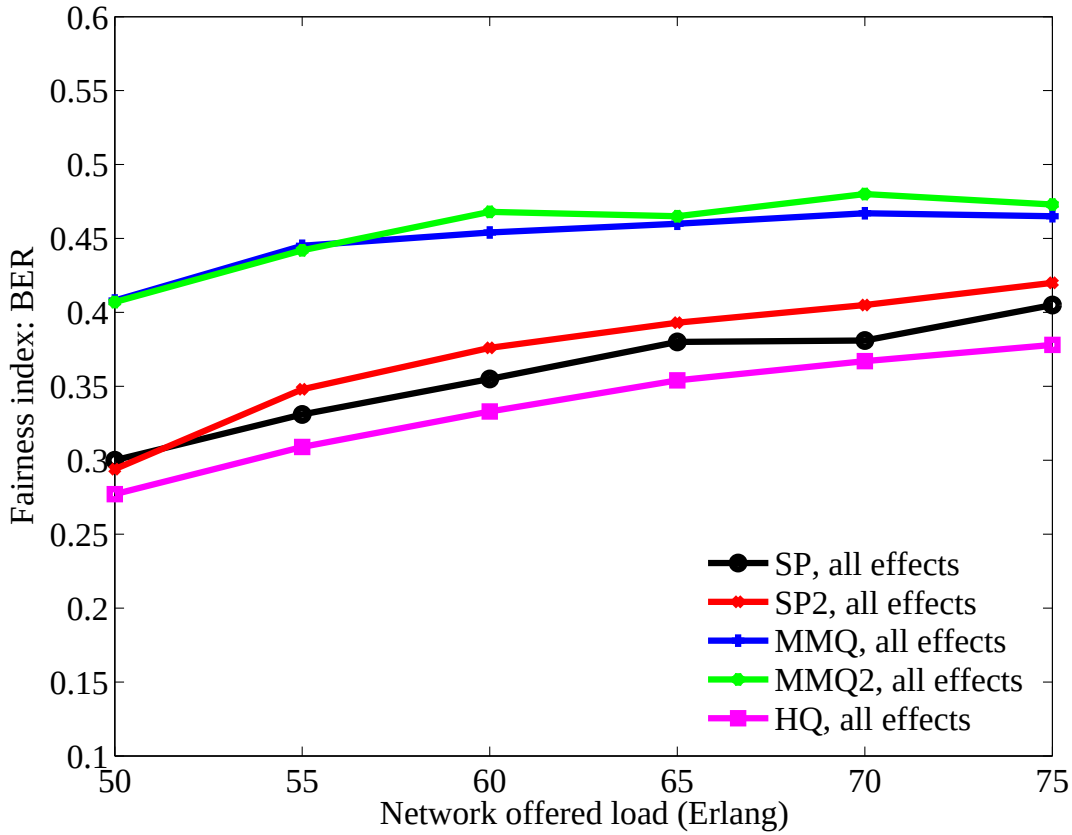
- ▷ Our QoS-enhanced RWA algorithms perform better for BER than SP/SP2

Fairness: Blocking Probability



▷ MMQ, SP2 exhibit highest BP fairness

Fairness: BER

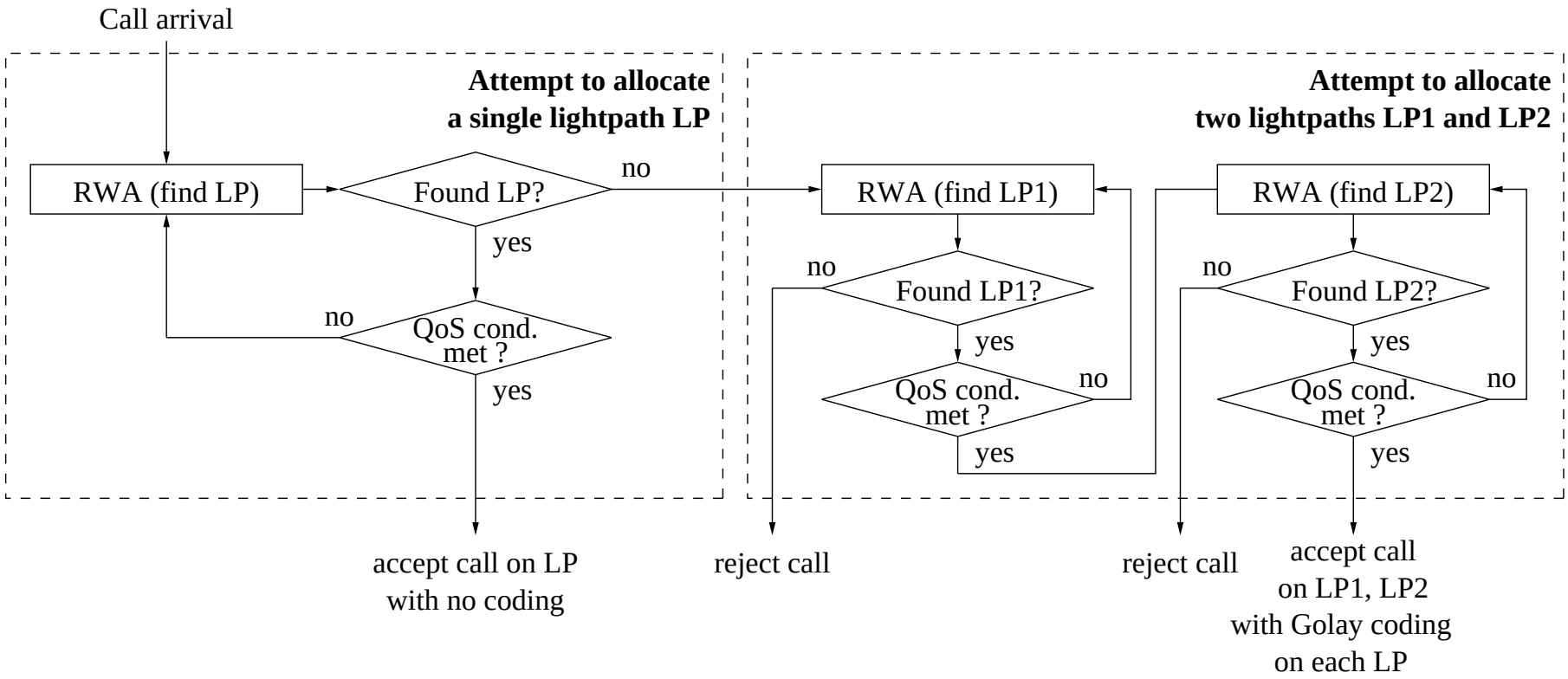


▷ MMQ exhibits highest BER fairness

RWA with optional coding for larger networks

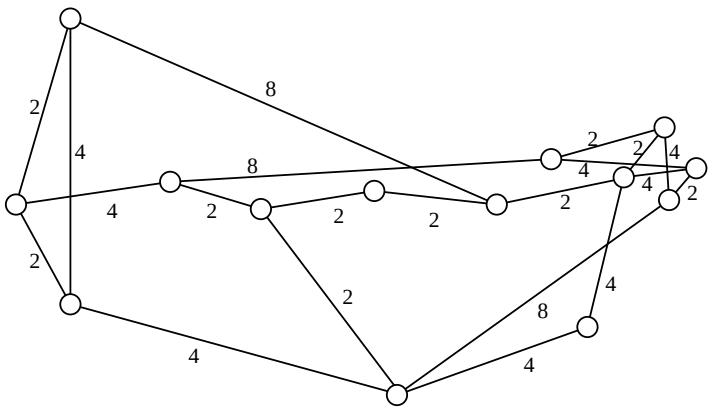
- Context: very large networks where some paths are "too" long
  - some paths are so long no call can be established without breaking the QoS constraint
- Forward Error Correction coding trade-off
  - can improve BER and keep it below threshold
  - cost: bandwidth expansion

Algorithm flow



- ▷ Golay (23, 12) code, rate $\approx 1/2$: need 2 LPs per call [Xu-PTL2005]
- ▷ RWA: adaptive routing, random wavelenth assignment

Simulation results

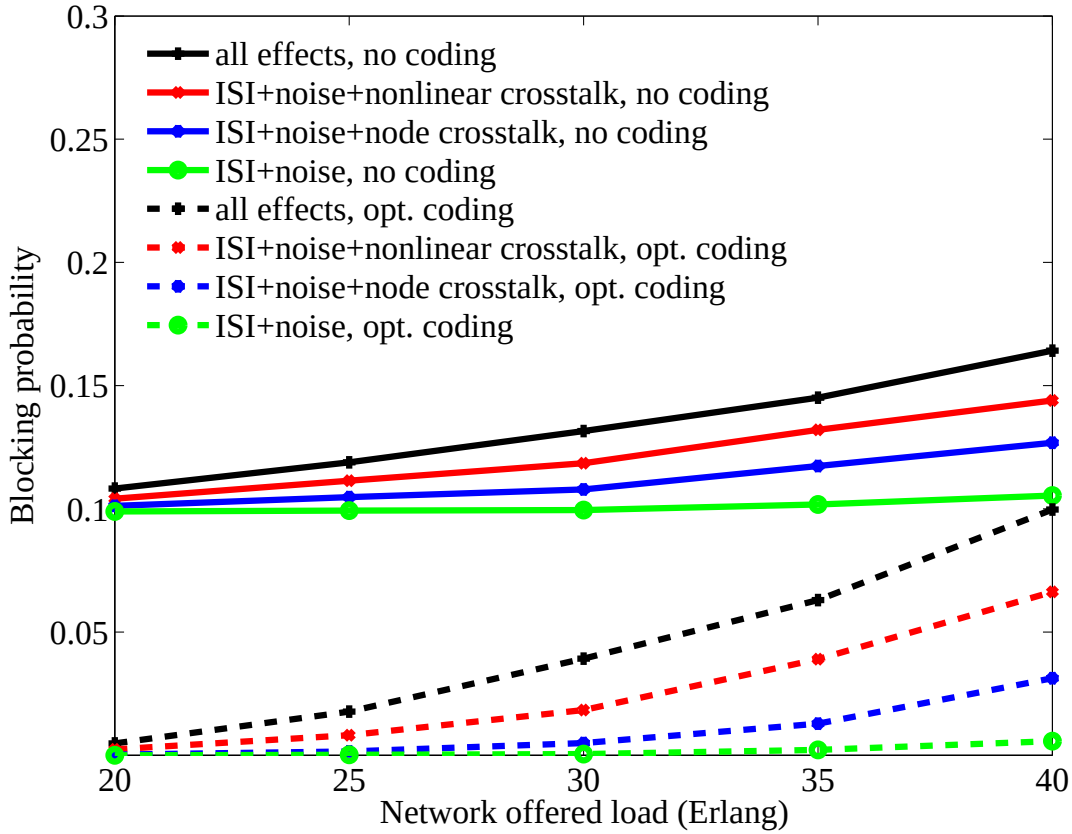


NSF topology downscaled 5 times; the weights are the number of spans.

Spans	2-12	14+
Paths	164	18

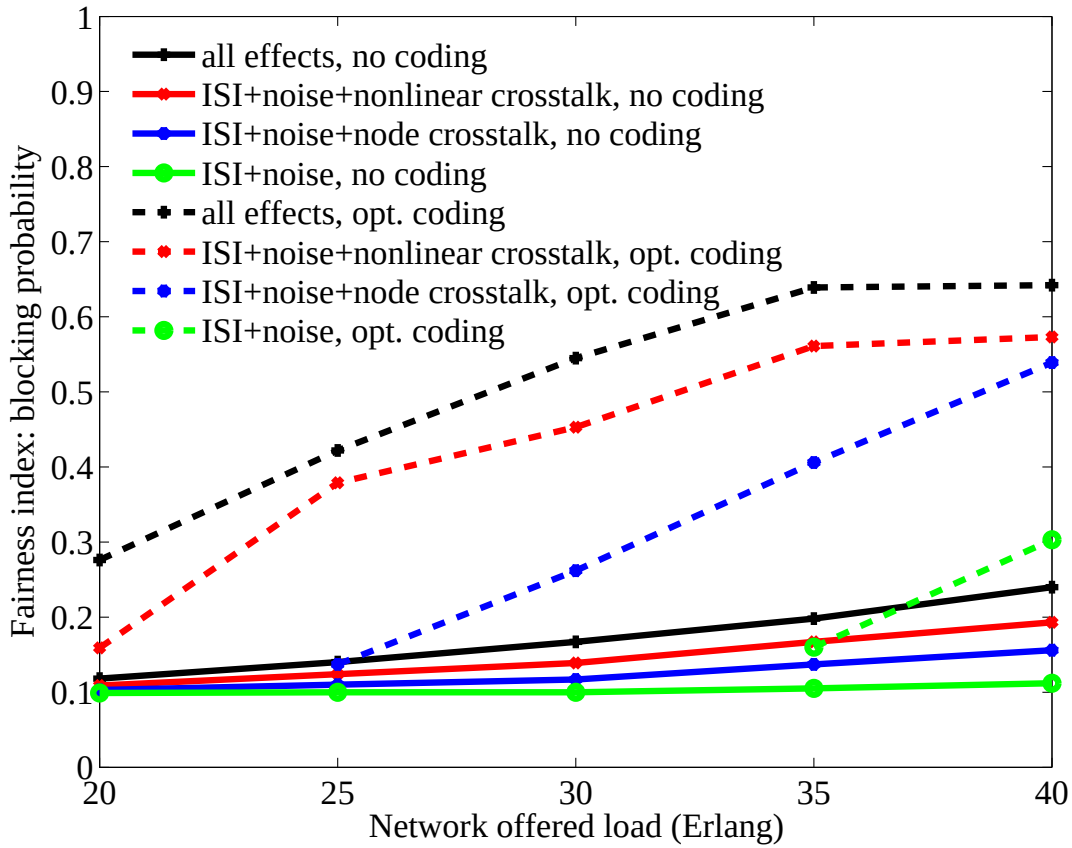
Description	**Value**
Span length	70 km
Signal peak power	2 mW
Bit rate	10 Gbps
Nonlinear parameter	2.2 $(\mathrm{W.m})^{-1}$
Pulse shape	NRZ
Fabric xtalk	−40 dB
Adj. port xtalk	−30 dB
Non adj. port xtalk	−60 dB
Fiber type	SMF
Dispersion compensation	100% post-DC
Number of WL	8
Minimum Q factor to ensure BER$< 10^{-9}$	$Q_1 = 6$ $Q_2 = 3.6$

Blocking probability



▷ Optional coding cancels most of the physical layer impairments

Fairness: blocking probability



▷ Optional coding improves fairness

Conclusions

- Introduced new metrics to evaluate RWA algorithms
- Large (metro/regional) networks: presented new QoS-enhanced RWA algorithms based on adaptive RWA: SP2, HQ, MMQ, MMQ2
  - Perform well in terms of BP
  - Perform better than others in terms of QoS and fairness
- Very large (regional) networks: optional coding ...
  - helps reducing blocking probabilities in large networks
  - mitigates physical layer impairments
  - improves fairness among users

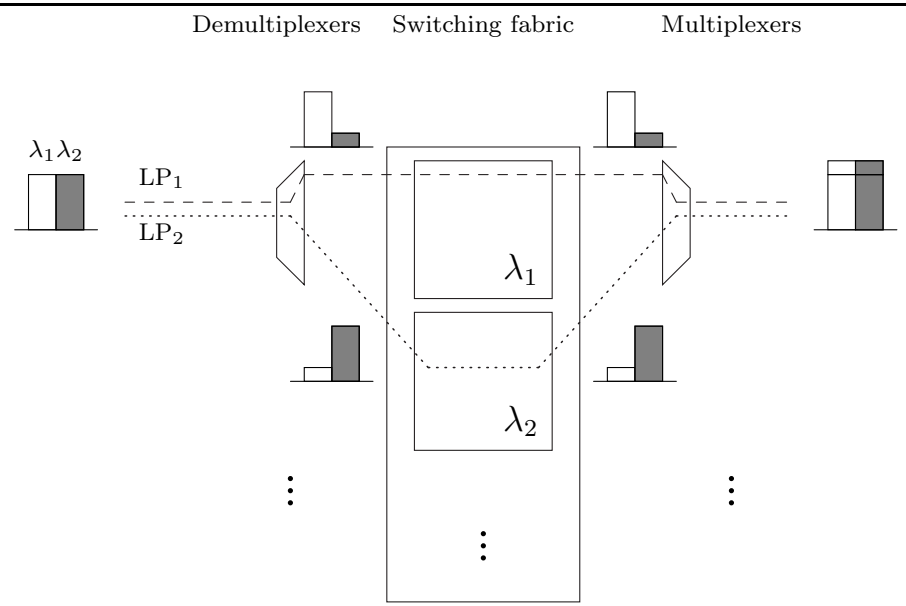
Overview

- Introduction
- Crosstalk Propagation in All-Optical Networks
- Cross-Layer Adaptive RWA in All-Optical Networks
- **Analysis of Blocking Probability in Crosstalk-Impaired Networks**
  - Assumptions
  - Iterative algorithm overview
  - QoS blocking
  - Validation by simulations
- Conclusions

Analysis of Blocking Probability in Crosstalk-Impaired Networks

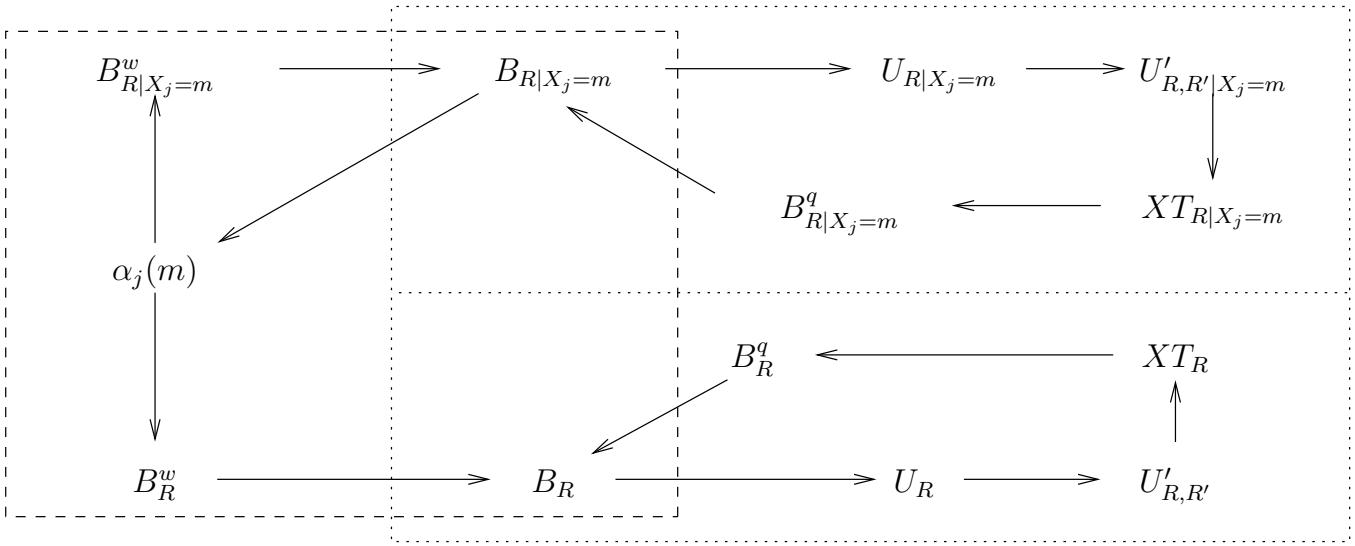
- Assumptions:
  - fixed routing, random pick wavelength assignment
  - ISI, noise, node demultiplexer crosstalk
  - wavelength equivalence
- Reuse published algorithm for blocking due to the wavelength continuity constraint (wavelength blocking) [Sridharan/Sivarajan-ToN2004]
- QoS extensions are largely independent of the wavelength blocking algorithm

Node crosstalk model [Deng/Subramaniam-Broadnets2004]



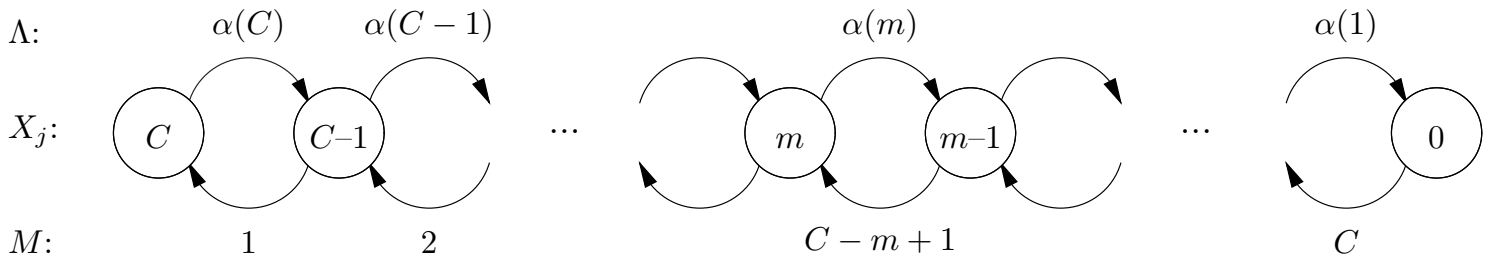
- ▷ "self-crosstalk"
- ▷ all channels are equivalent
  - no distinction between adjacent and non-adjacent channels

Iterative technique



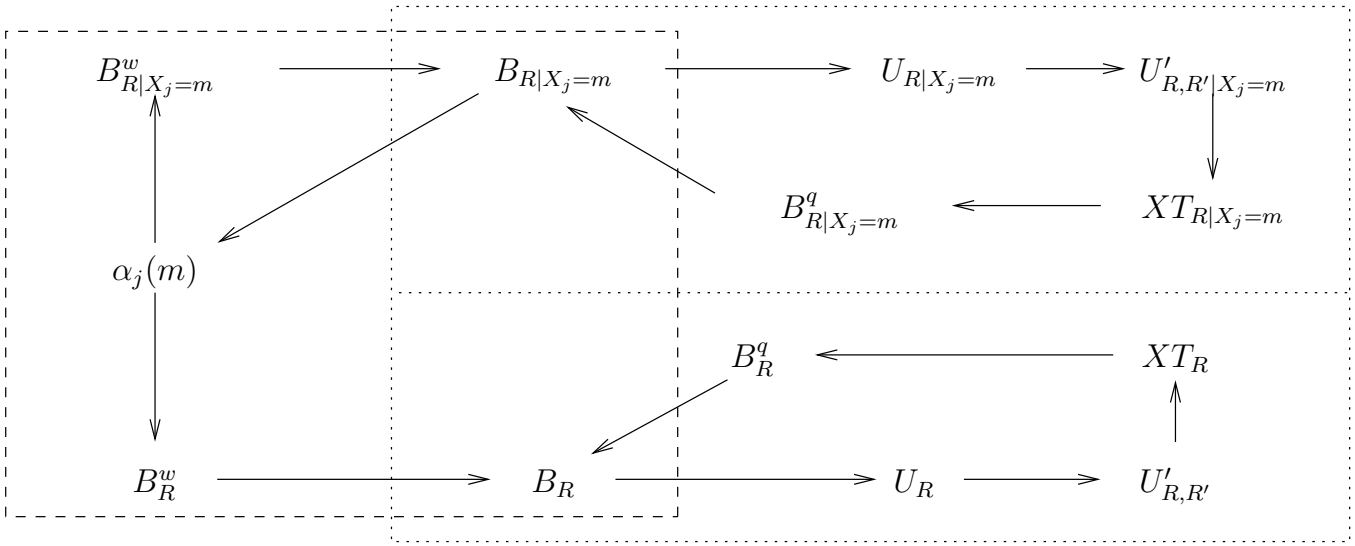
- ▷ Dashed box: wavelength blocking – Dotted box: QoS blocking
- ▷ X_j = number of free wavelengths on link j
- ▷ B_R, B^w_R, B^q_R are blocking probabilities ($B_R = B^w_R + (1 - B^w_R)B^q_R$)
- ▷ $\alpha_j(m)$ = state dependent arrival rate
- ▷ U, U', XT to be detailed

Per-link arrivals



- ▷ Birth-death process is known to accurately model link states [Chung/Kasper/Ross-ToN1993]
- ▷ X_j = number of free wavelengths on link j
- ▷ Λ = arrival rate (parameter of a Poisson distribution)
- ▷ M = service rate (parameter of an exponential distribution)

Iterative technique



- ▷ B^w_R and B^q_R are interdependent
- ▷ The respective algorithms to compute B^w_R and B^q_R, though, are largely independent
- ▷ There are techniques in the literature to compute B^w_R
  - ■ We reuse one of them to determine B^q_R

QoS blocking

$$B_R \rightarrow U_R \rightarrow U'_{R,R'} \rightarrow XT_R \rightarrow B^q_R \rightarrow B_R$$

- $U_R(k)$ = probability that $k = 0, \ldots, C$ calls are established on **exactly** route R
- Establishing a call on R is modeled as a Bernoulli trial with success p_R:

$$p_R = \frac{\Lambda_R}{M_R}\frac{1 - B_R}{C} = \Lambda_R \frac{1 - B_R}{C}$$

- Assuming independence between the trials:

$$U_R(k) \approx \binom{C}{k} p_R^k (1 - p_R)^{C-k}, k = 0, 1, \ldots, C.$$

QoS blocking

$$B_R \rightarrow U_R \rightarrow U'_{R,R'} \rightarrow XT_R \rightarrow B^q_R \rightarrow B_R$$

- $n_{xt}(R, R')$ = number of common nodes between routes R and R' where crosstalk can occur
  - Recall: crosstalk from R' to R can occur at a node N when R and R' share the link before N and the link after N
- $U'_{R,R'}(k)$ = probability that route R' injects k crosstalk components on route R

$$U'_{R,R'}(n_{xt}(R, R')k) = U_{R'}(k)$$

- $XT_R(k)$ = probability that route R is subject to exactly k crosstalk components

$$XT_R = U'_{R,R_1} * \ldots * U'_{R,R_p}$$

QoS blocking

$$B_R \rightarrow U_R \rightarrow U'_{R,R'} \rightarrow XT_R \rightarrow B_R^q \rightarrow B_R$$

- Q factor for a system with crosstalk:

$$Q_R = \frac{\mu_{1,R} - \mu_{0,R}}{\sigma_{0,R} + \sqrt{\sigma_{i,R}^2 + \sigma_{n,R}^2 + n\sigma_{x,R}^2}}$$

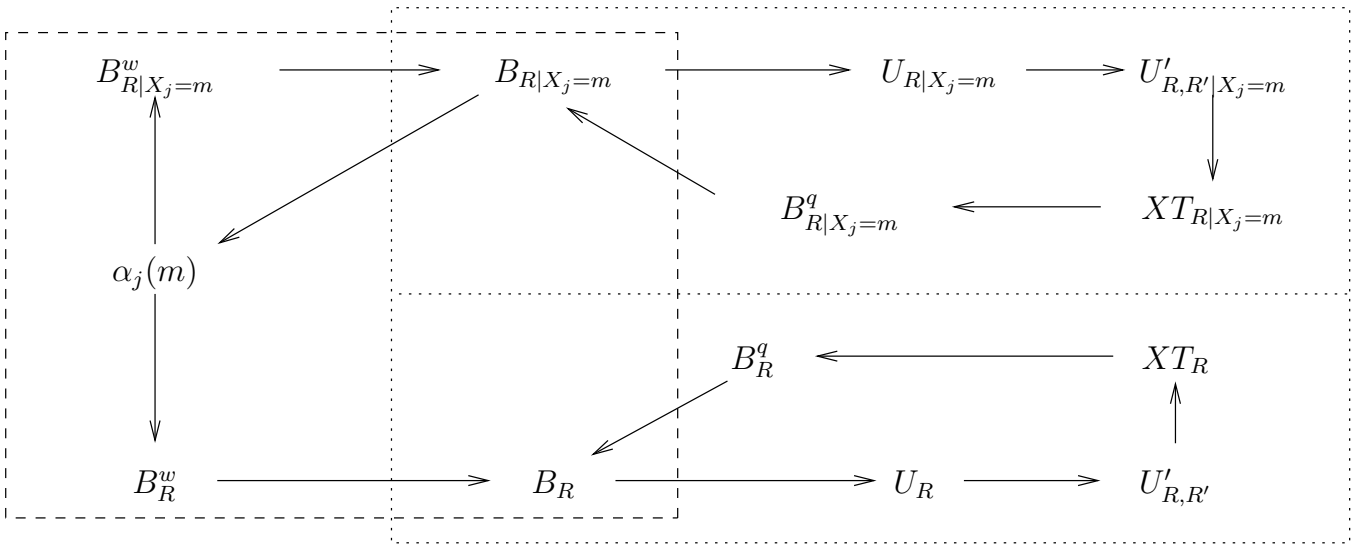
- Maximum number of crosstalks to keep Q below threshold:

$$N_R^{\max} = \left\lfloor \frac{\left(\frac{\mu_1 - \mu_0}{Q_{th}} - \sigma_{0,R}\right)^2 - \sigma_{i,R}^2 - \sigma_{n,R}^2}{\sigma_{x,R}^2} \right\rfloor$$

- QoS blocking:

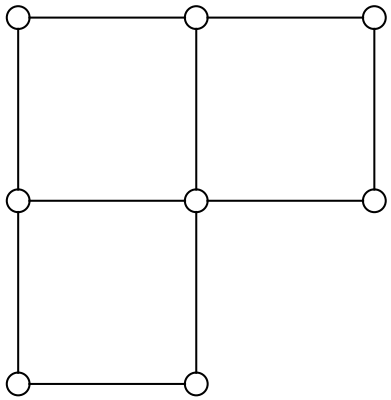
$$B_R^q = \sum_{k > N_R^{\max}} XT_R(k).$$

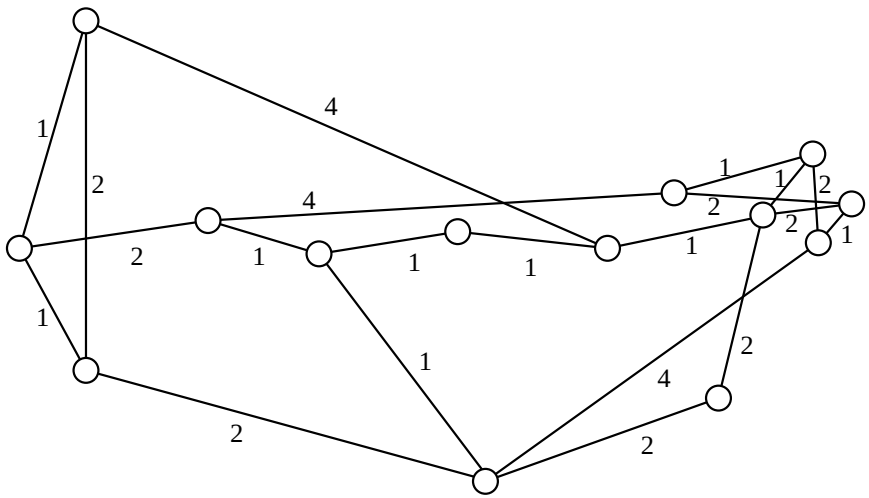
Iterative technique



- ▷ Computations of the conditionals $U_{R|X_j=m}$, $U'_{R,R'|X_j=m}$, $XT_{R|X_j=m}$, $B^q_{R|X_j=m}$ are very similar to the computations of U_R, $U'_{R,R'}$, XT_R, B^q_R.

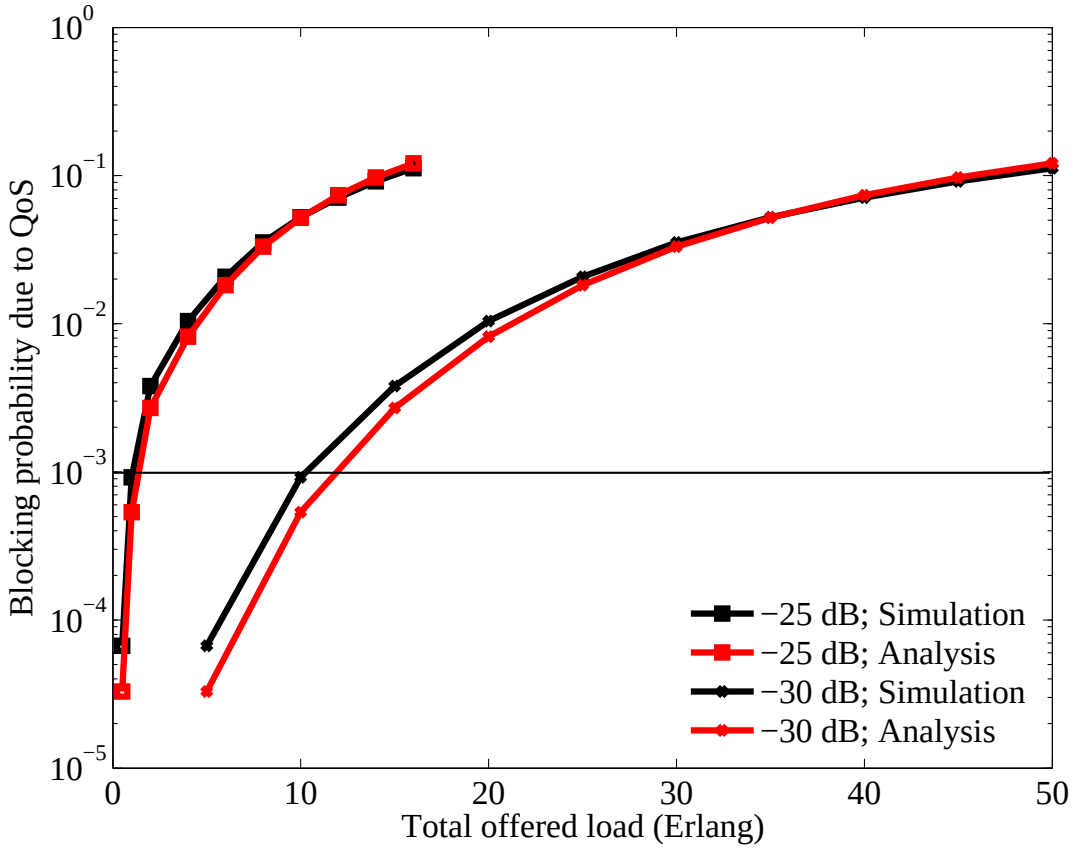
Simulation results





Description	Value
Span length	70 km
Signal peak power	2 mW
Bit rate	10 Gbps
Nonlinear parameter	2.2 $(\mathrm{W.m})^{-1}$
Pulse shape	NRZ
Fiber type	SMF
Dispersion	100% post-DC
Min. Q factor	6

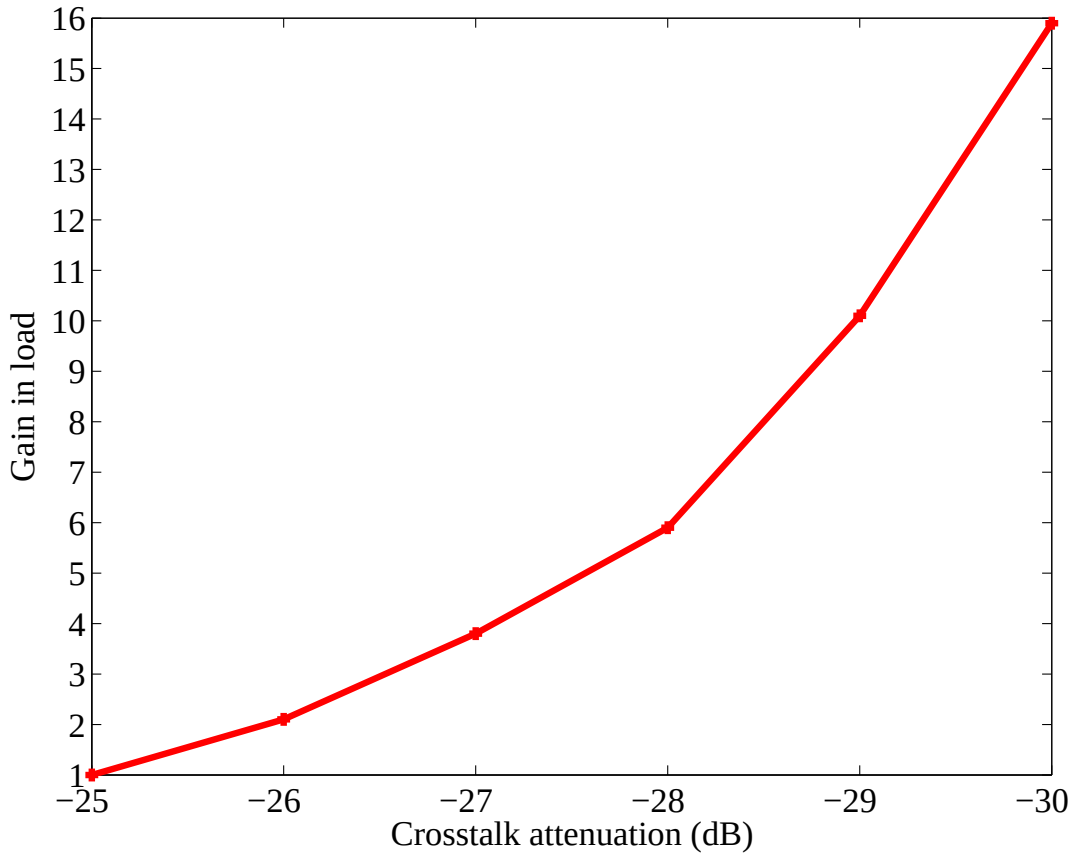
Simulation results



▷ mesh of 8 nodes, 16 wavelengths, -25 dB and -30 dB crosstalk

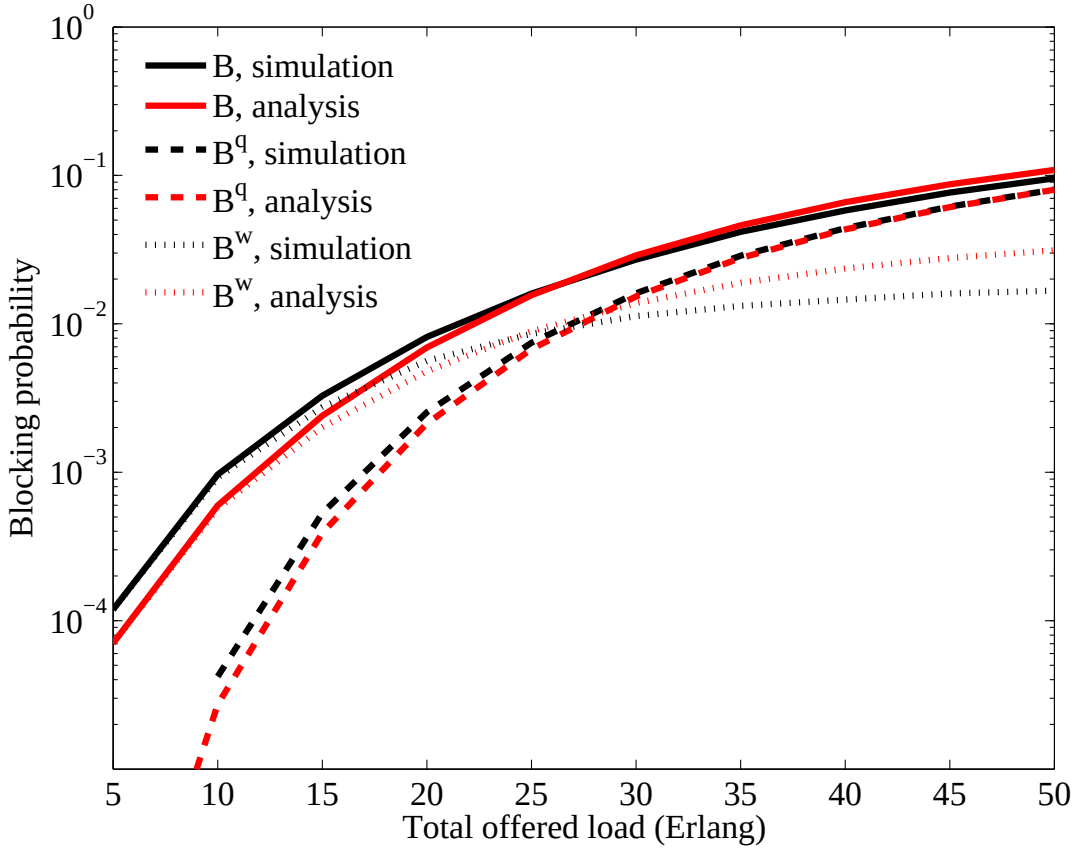
red: analysis; **black**: simulation

Simulation results



- ▷ Load in gain for a target average call blocking probability of 0.001 (reference: gain=1 for -25 dB)

Simulation results



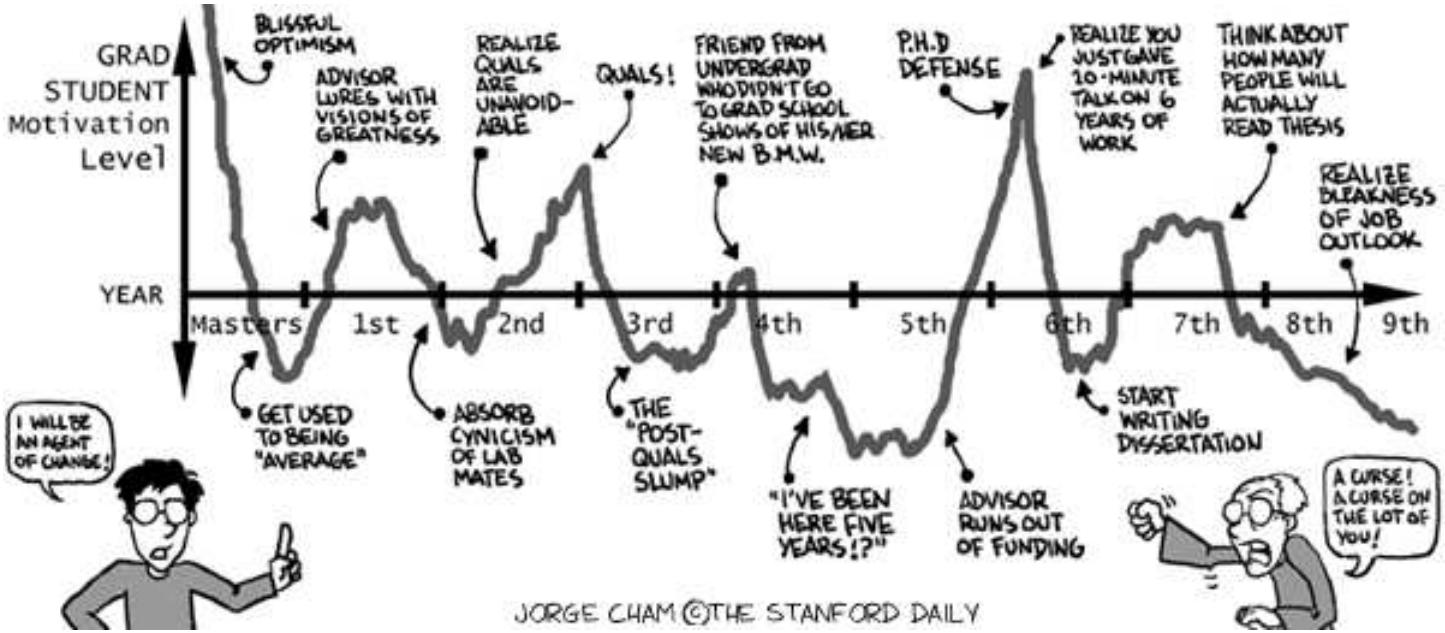
▷ NSF topology, 8 wavelengths, -30 dB crosstalk

red: analysis; **black**: simulation

Conclusions and future work

- Physical parameters-dependent impact of crosstalk on network performance
- Physical layer impairments (including crosstalk) mitigation is possible through RWA
- Cross-layer RWA analysis
- Leads for future work:
  - Higher speeds: Polarization Mode Dispersion (PMD) an issue for 40 Gbps
  - Wavelength regeneration, translucent networks
  - Adjacent vs. non-adjacent channel crosstalk in RWA analysis

Questions ?



["Piled Higher and Deeper" by Jorge Cham - www.phdcomics.com]

Contents of the transfer matrix [Xu-PTL2004]

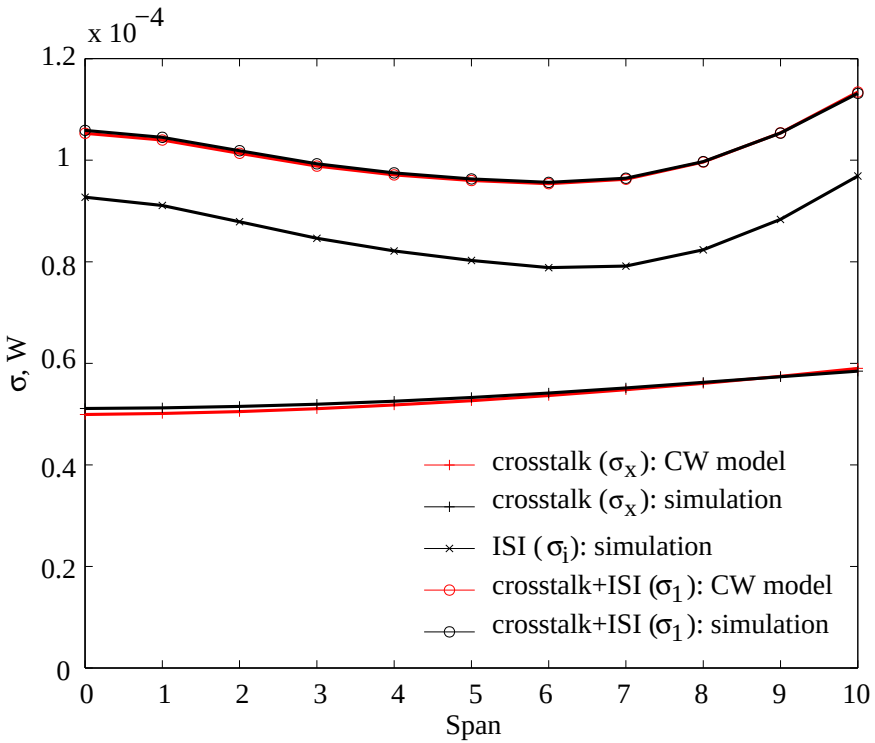
$$T_k(\omega) = \underbrace{\begin{bmatrix} \frac{1}{2}e^{-j\theta_{SPM}} & \frac{1}{2}e^{j\theta_{SPM}} \\ \frac{1}{2j}e^{-j\theta_{SPM}} & -\frac{1}{2j}e^{j\theta_{SPM}} \end{bmatrix}}_{\text{SPM}} \underbrace{\mathcal{D}(\omega)}_{\text{disp. comp.}} \underbrace{\mathcal{M}_k(\omega) \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix}}_{\text{disp. and NL}}$$

$$\mathcal{M}_k(\omega) = \begin{bmatrix} \mathcal{M}_{1,1}(\omega) & \mathcal{M}_{1,2}(\omega) \\ \mathcal{M}^*_{1,2}(\omega) & \mathcal{M}^*_{1,1}(\omega) \end{bmatrix}$$

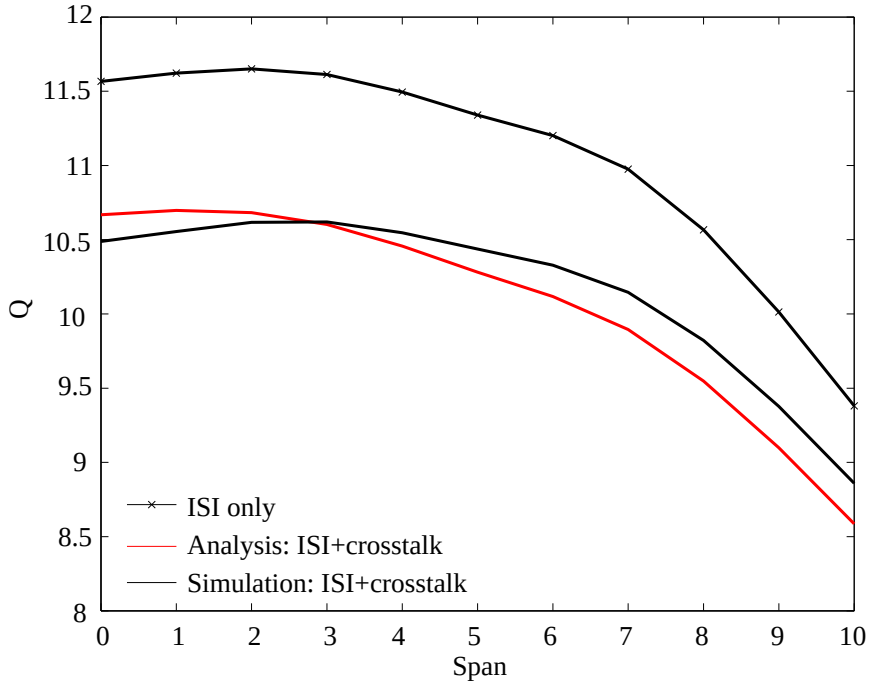
$$\mathcal{M}_{1,1}(\omega) = e^{-\frac{\alpha}{2}L} G e^{-\frac{j}{2}(\beta_2 L)\omega^2} \left(1 - j\frac{2\gamma P_0}{\alpha} + \frac{\gamma^2 P_0^2}{2\alpha(\alpha - j\beta_2\omega^2)} - \frac{2\gamma^2 P_0^2}{\alpha^2}\right)$$

$$\mathcal{M}_{1,2}(\omega) = e^{-\frac{\alpha}{2}L} G e^{-\frac{j}{2}(\beta_2 L)\omega^2} \left(-j\frac{\gamma P_0}{\alpha - j\beta 2\omega^2} - \frac{2\gamma^2 P_0^2}{\alpha(2\alpha - j\beta_2\omega^2)}\right)$$

NRZ - 2.5 Gbps non dispersion-compensated network

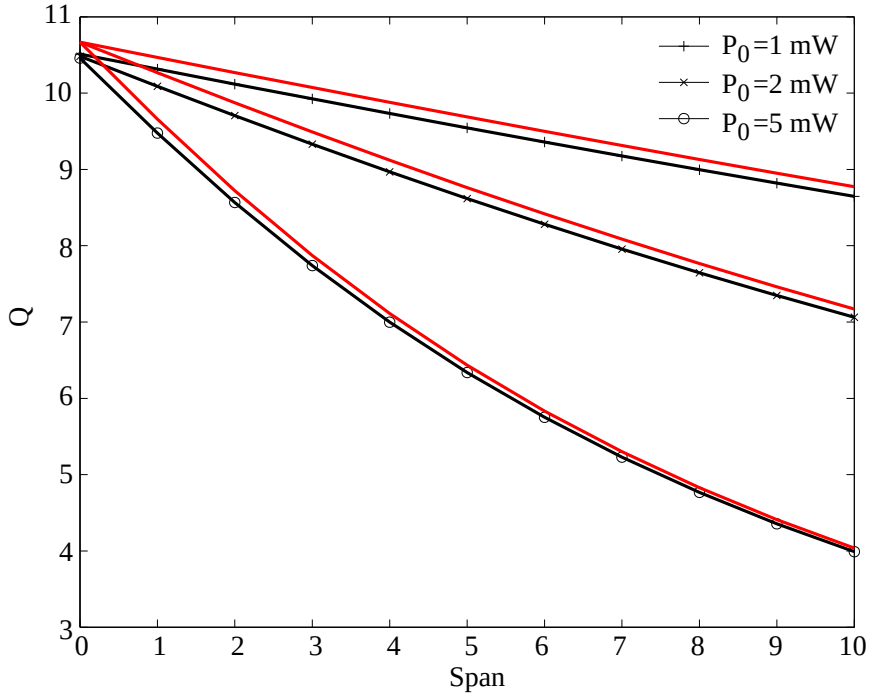


Standard deviations

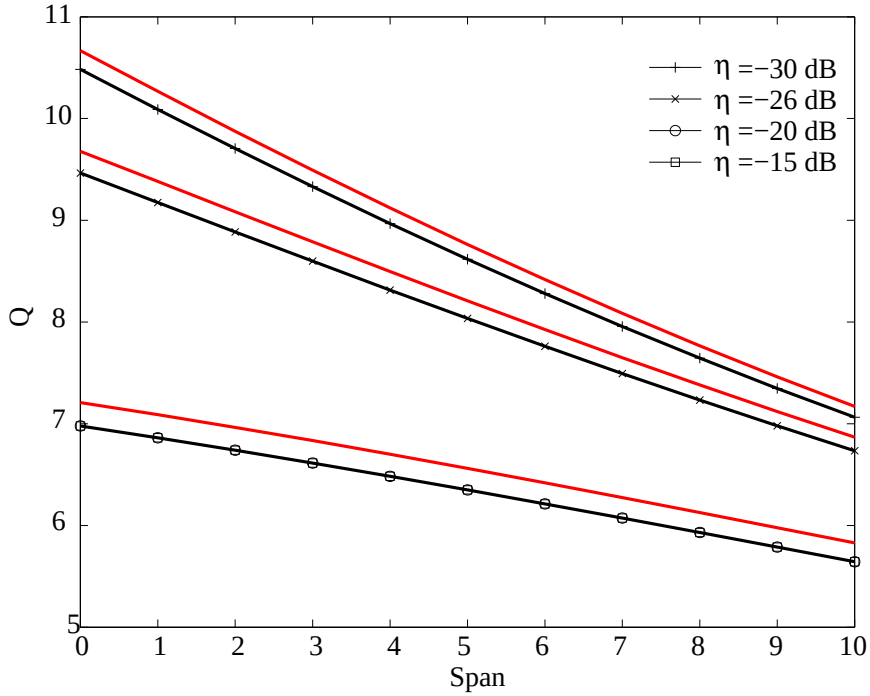


Q factors

NRZ - Main signal power and node crosstalk attenuation



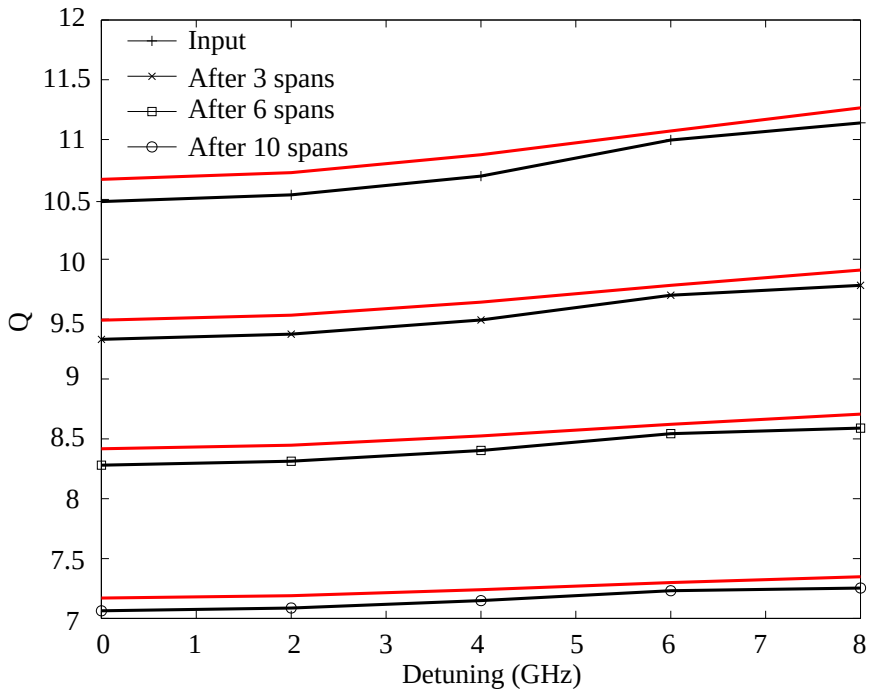
Main signal power



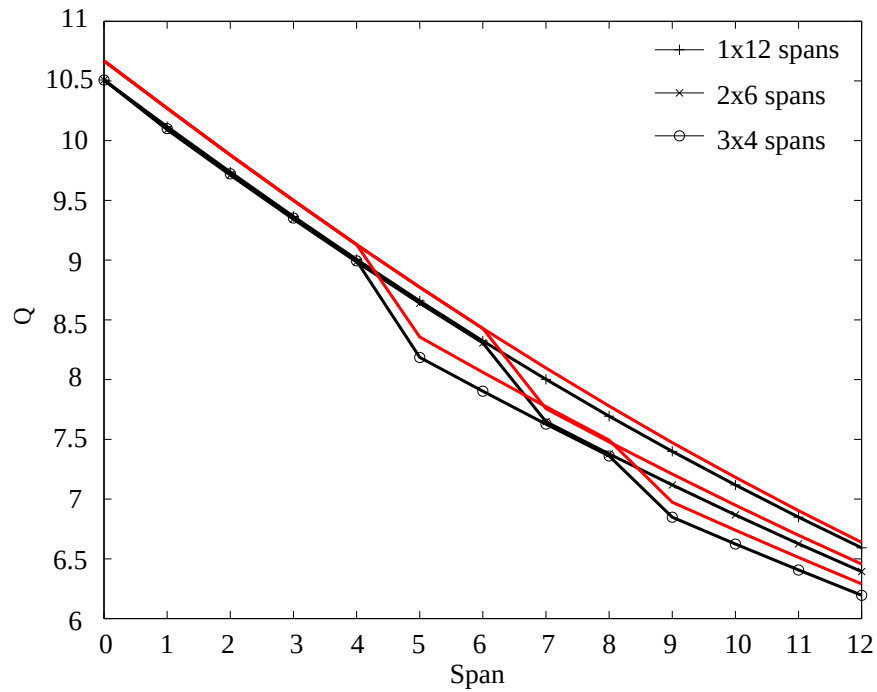
Node crosstalk attenuation

red: analysis; **black**: simulation

NRZ - Detuning and multisegment



Detuning



Multisegment

red: analysis; **black**: simulation

Semi-analytical method for RZ signals

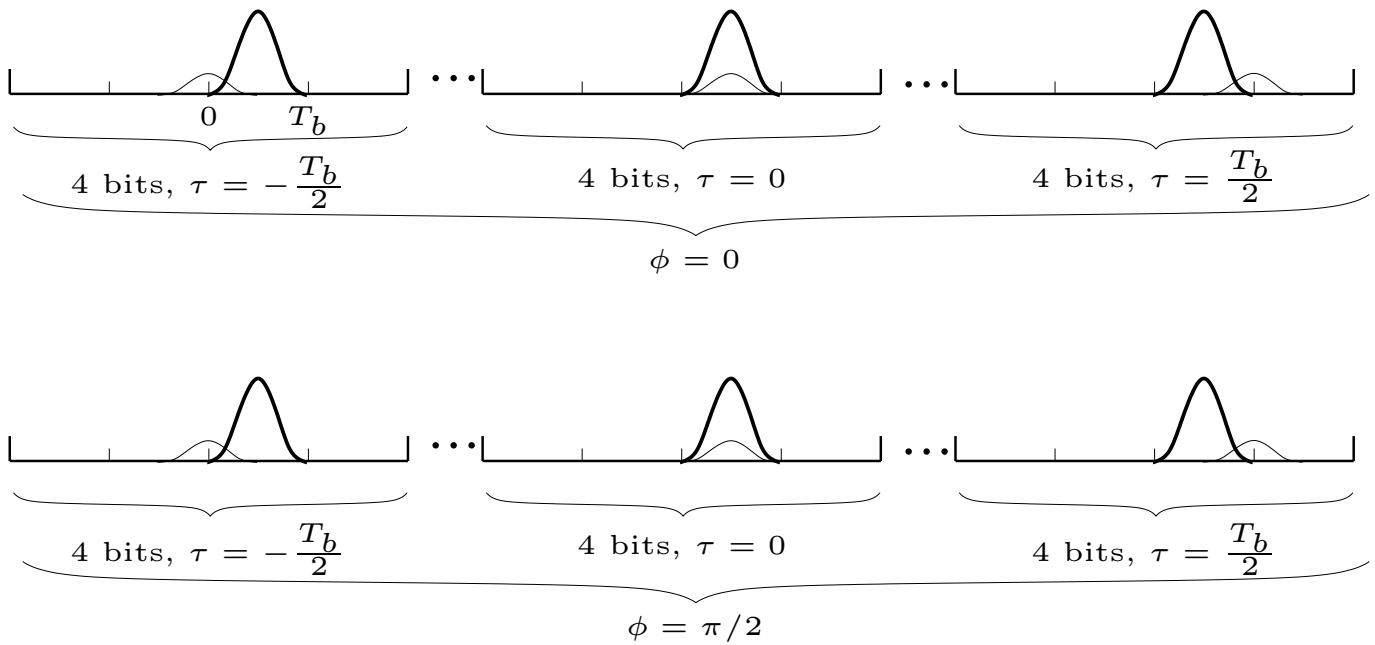
- For a single main bit and node crosstalk bit — output:

$$s_{out}(t) = \sqrt{P(t)}\exp\left(j\theta(t)\right) + m g_{\tau,\varphi}(t)$$

- Splitting as in the CW case:

$$\begin{aligned} s_{out}(t) = \exp\left(j\theta(t)\right)\Bigg(&\sqrt{P(t)} + \\ &m\left(g_\tau^{I+}(t)\exp\left(j\varphi\right) + g_\tau^{I-}(t)\exp\left(-j\varphi\right)\right. \\ &\left.+ j g_\tau^{Q+}(t)\exp\left(j\varphi\right) + j g_\tau^{Q-}(t)\exp\left(-j\varphi\right)\right)\Bigg) \end{aligned}$$

Semi-analytical method



- Use short simulations, use N values for the random delay τ: $\tau_k = k/NT_b$, and 2 phases: $\phi = 0$ and $\phi = \pi/2$

Continued ...

- Short simulation output:

$$s_{k,\phi}^{out}(t) \approx \sqrt{P(t)} \exp j\theta(t) + g_{\tau_k,\phi}(t)$$

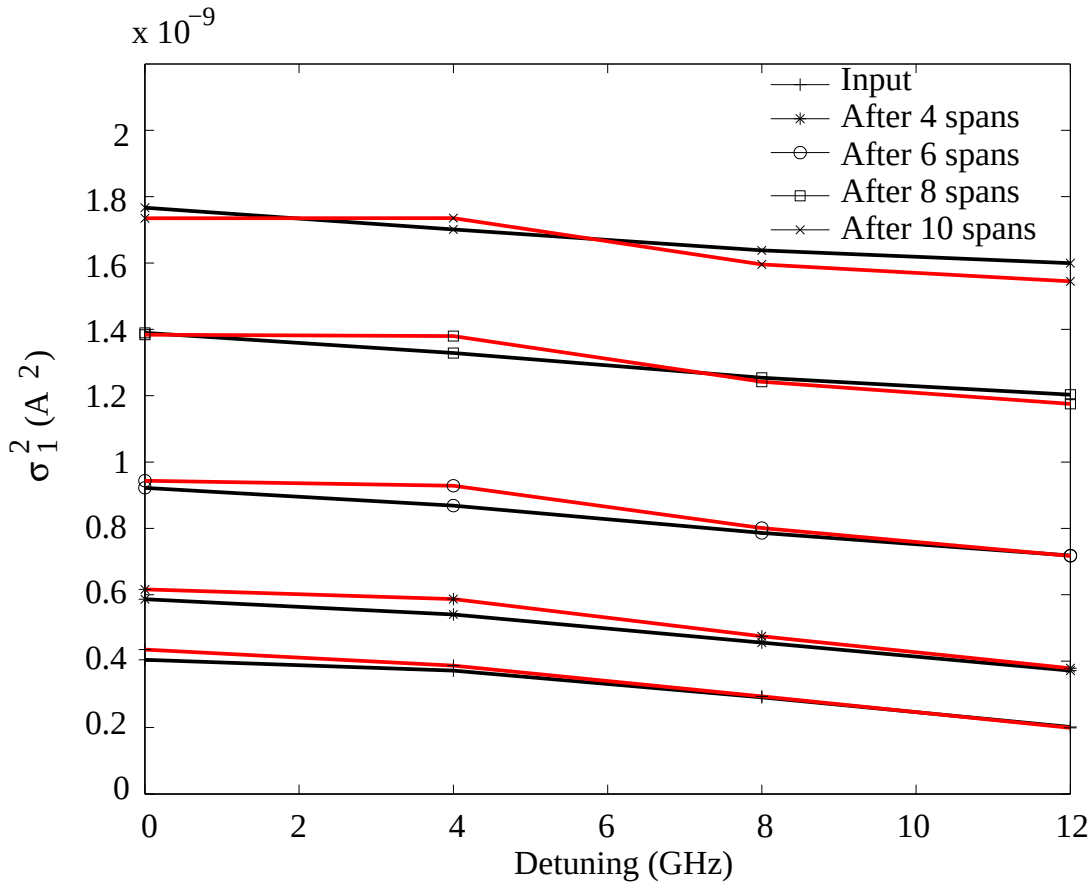
- Estimate of the node crosstalk pulse shape for τ_k

$$\begin{aligned}\hat{g}_{\tau_k}^{I+}(t) &= \frac{1}{2}\Re\left\{\exp\left(-j\theta(t)\right)(s_{k,0}^{out}(t) - \sqrt{P(t)}\exp j\theta(t))\right\} \\ &\quad -\frac{j}{2}\Re\left\{\exp\left(-j\theta(t)\right)(s_{k,\frac{\pi}{2}}^{out}(t) - \sqrt{P(t)}\exp j\theta(t))\right\}\end{aligned}$$

- Estimate for the node crosstalk variance:

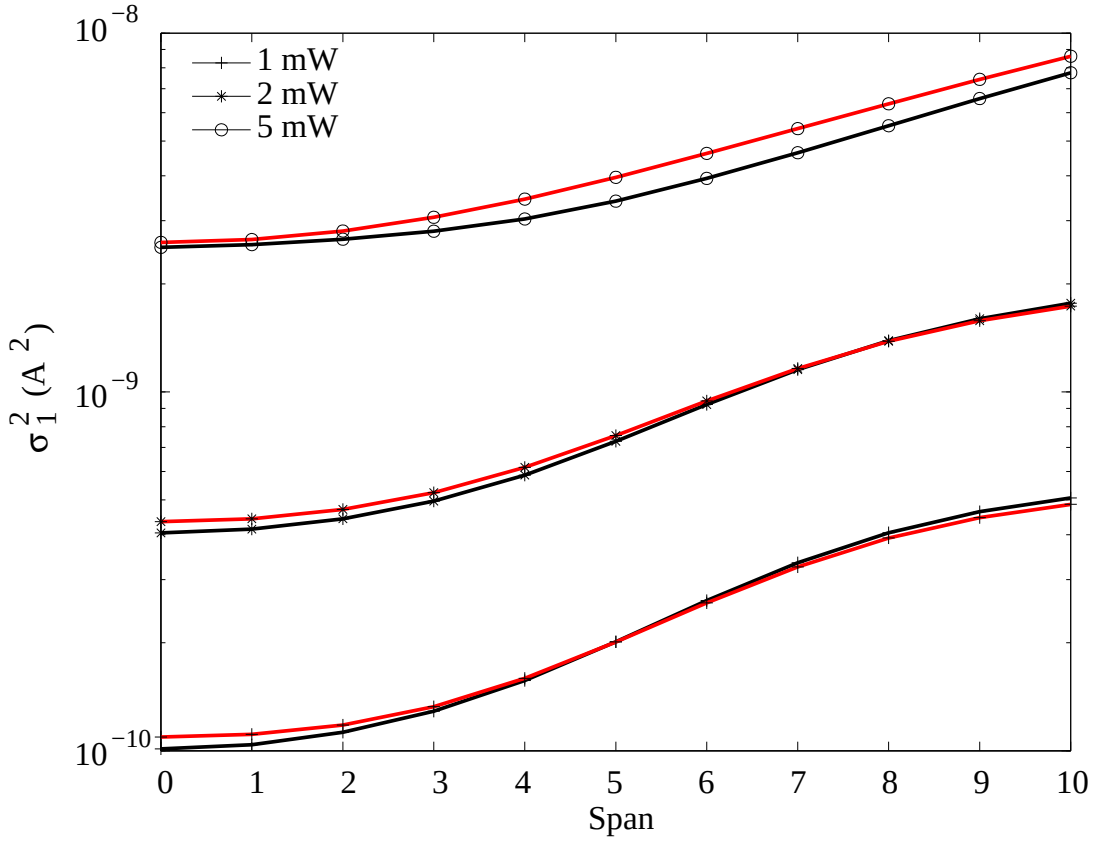
$$\hat{\sigma}_x^2 = \left(\frac{\rho^2}{N}\sum_{k=-N/2}^{N/2-1} 4\left|f(t) * \left(\sqrt{P(t)}\hat{g}_{\tau_k}^{I+}(t)\right)\right|^2\right)\Bigg|_{t=T_b/2}$$

RZ - Impact of detuning



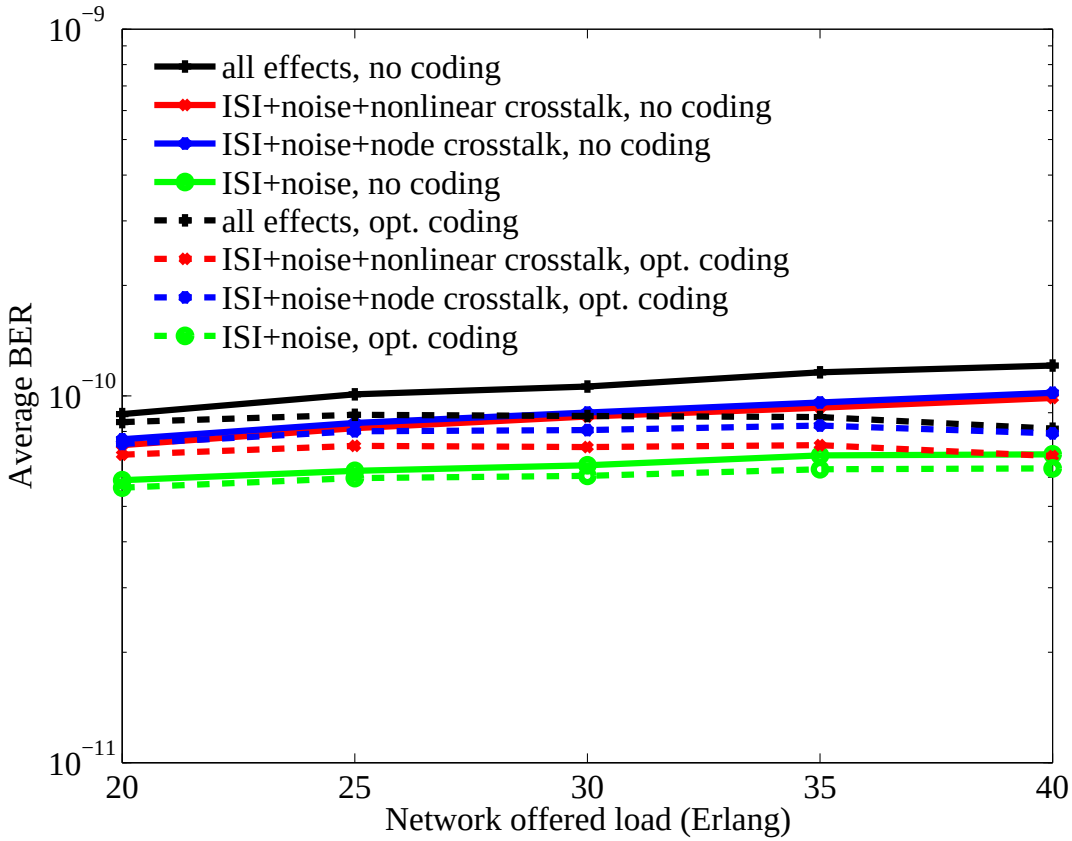
red: analysis; **black**: simulation

RZ - Impact of main signal power



red: analysis; **black**: simulation

RWA with optional coding: BER



▷ Optional coding lowers BER